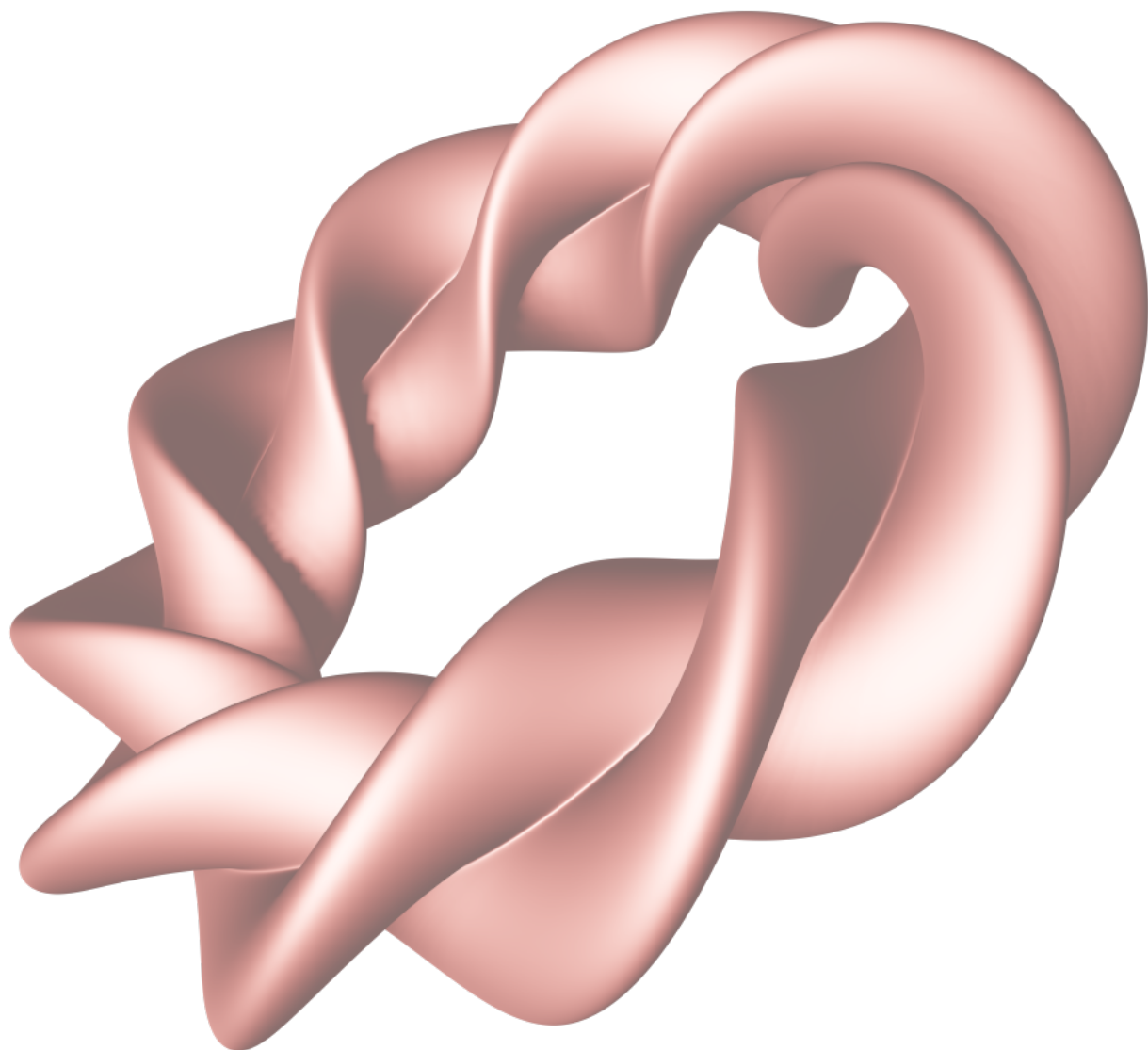


# GLOBAL ANALYSIS ON MANIFOLDS



*Programme*

19-23 September 2022

	Monday – 19/09/2022	Tuesday – 20/09/2022	Wednesday – 21/09/2022	Thursday – 22/09/2022	Friday – 23/09/2022
09:00		Lars Andersson <i>Gravitational instantons and special geometry</i>	Werner Ballmann <i>Essential spectrum and small eigenvalues</i>	Gerhard Huisken <i>Foliations in asymptotically flat 3-manifolds</i>	Dorothee Schüth <i>On the Dirac spectrum of homogeneous 3-spheres</i>
10:15	REGISTRATION and COVFEFE	COVFEFE	COVFEFE	COVFEFE	COVFEFE
11:00	Bernhard Hanke <i>Lipschitz rigidity for scalar curvature</i>	Nadine Große <i>Boundary values on domains with cusps</i>	Klaus Fredenhagen <i>Causal structure of spacetime and construction of quantum field theory theory</i>	Gilles Carron <i>Euclidean heat kernel rigidity and applications</i>	Paolo Piazza <i>Analysis and Geometry of Dirac operators on singular spaces</i>
12:00			LUNCH		
14:00	Dorothea Bahns <i>On invariants for minimal surfaces and strings</i>	Sylvie Paycha <i>Mathematical reflections on locality</i>		Rafe Mazzeo <i>The Extended Bogomolny Equations</i>	
15:15	COVFEFE	COVFEFE		COVFEFE	
16:00	Mattias Dahl <i>Scalar curvature rigidity of Einstein Manifolds</i>	Thomas Leistner <i>The range of a connection and a Calabi operator for locally symmetric spaces</i>		Oliver Lindblad Petersen <i>Wave equations in subextremal Kerr-de Sitter spacetimes</i>	
17:15	Chris Fewster <i>Green hyperbolicity for nonlocal operators</i>	Alexander Strohmaier <i>Index theory on spacetimes</i>		Matthias Ludewig <i>The stringor bundle of a string manifold</i>	
18:30				CONFERENCE DINNER	



**10:15 – 11:00**

**Registration and Coffee**

(Aula, KG I, Platz der Universität 3, 79098 Freiburg)

**11:00 – 12:00**

**Bernhard Hanke (Augsburg)**

(Aula, KG I, Platz der Universität 3, 79098 Freiburg)

*Lipschitz rigidity for scalar curvature*

Lower scalar curvature bounds on spin Riemannian manifolds exhibit remarkable rigidity properties determined by spectral properties of Dirac operators. For instance, a fundamental result of Llarull states that there is no smooth Riemannian metric on the  $n$ -sphere which dominates the round metric and whose scalar curvature is greater than or equal to the scalar curvature of the round metric, except the round metric itself. A similar result holds for smooth comparison maps from spin Riemannian manifolds to round spheres.

In a joint work with Simone Cecchini and Thomas Schick, we generalize this result to Riemannian metrics with regularity less than  $C^1$  and Lipschitz comparison maps, answering a question of Gromov in his "Four Lectures". To this end, we rely on a notion of scalar curvature in the distributional sense introduced by Lee-LeFloch and on spectral properties of Lipschitz Dirac operators. It turns out that the existence of a nonzero harmonic spinor field - guaranteed by the Atiyah-Singer index theorem - forces the given comparison map to be quasiregular in the sense of Reshetnyak. Thus we build an unexpected bridge from spin geometry to the theory of quasiconformal mappings.

**12:00 – 14:00**

**Lunch**

**14:00 – 15:00**

**Dorothea Bahns (Göttingen)**

(Aula, KG I, Platz der Universität 3, 79098 Freiburg)

*On invariants for minimal surfaces and strings*

I will report on Pohlmeier's theory of (diffeomorphism) invariants for minimal surfaces and world sheets of strings. The original construction, dating back to 1982, based on a certain connection, is slightly reformulated, its relation to the theory of integrable systems is highlighted and it is explained how the invariants are extracted from the coefficients of the connection. Moreover, a reconstruction theorem of Pohlmeier and Rehren is recounted and some explicit examples are given (partly due to Kornhäß). I will then explain how the Poisson algebra which the invariants form can be explained purely in terms of a certain Hopf algebra. This Poisson algebra serves as the starting point for a diffeomorphism invariant algebraic quantization by deformation (based on work by Pohlmeier, Rehren and Meusburger). During the talk, I hope to be able to point out how all this relates to Christian Bär.

This is based on work of the above mentioned authors and on joint work with V. Wolff and J. Meinecke.

**15:15 – 16:00**

**Coffee**

**16:00 – 17:00**

**Mattias Dahl (KTH Stockholm)**

(Aula, KG I, Platz der Universität 3, 79098 Freiburg)

*Scalar curvature rigidity of Einstein manifolds*

An Einstein manifold is called scalar curvature rigid if there are no compactly supported volume-preserving deformations of the metric  $g$  which increase the scalar curvature. We give characterizations of scalar curvature rigidity for open Einstein manifolds as well as for closed Einstein manifolds. As an application, we construct mass-decreasing perturbations of the Riemannian Schwarzschild metric and the Taub-Bolt metric.

This is joint work with Klaus Kröncke.

**17:15 – 18:15**

**Chris Fewster (York)**

(Aula, KG I, Platz der Universität 3, 79098 Freiburg)

*Green hyperbolicity for nonlocal operators*

One of Christian Bär's achievements has been the isolation and study of Green hyperbolic operators: partial differential operators on globally hyperbolic manifolds that, together with their formal adjoints, admit advanced and retarded Green operators. Bär has shown how substantial analytical detail may be extracted from this fairly minimal, essentially algebraic, input.

In this talk, I describe a generalisation of Green hyperbolicity to include operators that are sums of partial differential operators and nonlocal operators. I will also explain some applications, including the description of Unruh-DeWitt detectors in quantum field theory and models of quantum fields propagating on noncommutative spacetime backgrounds.

The talk is based partly on joint work with Rainer Verch, in preparation.

**09:00 – 10:00**

Lars Andersson (AEI Potsdam)

(Aula, KG I, Platz der Universität 3, 79098 Freiburg)

*Gravitational instantons and special geometry*

Gravitational instantons are Ricci flat complete Riemannian 4-manifolds with at least quadratic curvature decay. Classical examples include the Taub-NUT and Euclidean Kerr instantons. A classification of half-flat instantons is known but the uniqueness problem remains open in general. In this talk I will present some recent results on the classification of  $S^1$ -symmetric instantons obtained using an identity of Israel-Robinson type and the  $G$ -signature theorem, together with recent results on instantons with special geometry.

**10:15 – 11:00**

**Coffee**

**11:00 – 12:00**

Nadine Große (Freiburg)

(Aula, KG I, Platz der Universität 3, 79098 Freiburg)

*Boundary values on domains with cusps*

We consider boundary value problems of the Laplacian with Dirichlet (or mixed) boundary conditions on domains with singularities. In two dimensions these singularities include also cusps. Our approach is by blowing up the singularities via a conformal change to translate the boundary problem to one on a noncompact manifold with boundary that is of bounded geometry and of finite width. This gives a natural geometric interpretation in the appearing weights and additional conditions needed to obtain well-posedness results.

This is joint work with Bernd Ammann (Regensburg) and Victor Nistor (Universite de Lorraine).

**12:00 – 14:00**

**Lunch**

**14:00 – 15:00**

Sylvie Paycha (Potsdam)

(Aula, KG I, Platz der Universität 3, 79098 Freiburg)

*Mathematical reflections on locality*

Starting from the principle of locality in quantum field theory, which states that an object is influenced directly only by its immediate surroundings, I will first briefly review some features of the notion of locality arising in physics and mathematics. These are then encoded in locality relations, given by symmetric binary relations whose graph consists of pairs of "mutually independent elements". Locality morphisms, namely maps that factorise on products of such pairs of elements, play a key role in the context of renormalisation in multiple variables. They include "locality evaluators", which we use to consistently evaluate meromorphic germs in several variables at their poles. I will report on recent joint work with Li Guo and Bin Zhang which gives a classification of locality evaluators on certain classes of algebras of meromorphic germs.

**15:15 – 16:00**

**Coffee**

**16:00 – 17:00**

**Thomas Leistner (Adelaide)**

**(Aula, KG I, Platz der Universität 3, 79098 Freiburg)**

*The range of a connection and a Calabi operator for locally symmetric spaces*

On a semi-Riemannian manifold, the Killing operator maps a vector field to the Lie derivative of the metric along the vector field. The kernel of this first order linear differential operator consists of the Killing vector fields and, as infinitesimal isometries, they usually are at the centre of interest. In the talk, however, we focus on the range of the Killing operator. For spaces of constant sectional curvature, Calabi found a second order linear differential operator that provides exact local integrability conditions for the range of the Killing operator. We generalise this result by providing such a second order operator for indecomposable locally symmetric spaces (Riemannian and Lorentzian) and, whenever possible, their products. Our approach uses the prolongation of the Killing operator to a connection, which leads us to analyse the range of a connection on a vector bundle in general. We find a sufficient condition for local exactness at the start of the complex that is obtained by dividing the twisted de Rham sequence by the range of the curvature. This criterion is used to prove the local exactness of our Calabi operator for locally symmetric spaces when it holds, and identify the products for which it fails.

This is joint work with Federico Costanza, Mike Eastwood and Ben McMillan.

**17:15 – 18:15**

**Alexander Strohmaier (Leeds)**

**(Aula, KG I, Platz der Universität 3, 79098 Freiburg)**

*Index theory on spacetimes*

I will review some general index theorems for Dirac type operators on Lorentzian spacetimes and explain the main differences to the theory in Riemannian signature. I will explain the relations between index theory and the Feynman propagator. The latter is unavoidable in quantum field theory in curved spacetimes and can also be linked to the eta invariant of a Cauchy hypersurface. If there is time I will give some applications in mathematical physics.

**09:00 – 10:00**

Werner Ballmann (Bonn)

(HS II, Albertstr. 23b, 79104 Freiburg)

*Essential spectrum and small eigenvalues*

I will discuss the notion of small eigenvalues and report on joint work with Panagiotis Polymerakis on essential spectrum and small eigenvalues of geometrically finite manifolds.

**10:15 – 11:00**

Coffee

**11:00 – 12:00**

Klaus Fredenhagen (Hamburg)

(HS II, Albertstr. 23b, 79104 Freiburg)

*Causal structure of spacetime and construction of quantum field theory*

The  $S$ -matrices corresponding to interactions supported in compact subregions of space-time generate  $C^*$ -algebras of local observables. They are subject to a few axioms involving the causal structure of spacetime and a classical Lagrangian. In addition symmetries of the classical configuration space are modified by a cocycle with values in a group of transformations of local interactions corresponding to the renormalisation group of formal perturbation theory. The arising system of  $C^*$ -algebras satisfies the axioms of locally covariant quantum field theory: it is a functor from a category of Lorentzian spacetimes, equipped with classical Lagrangians, to the category of  $C^*$ -algebras. It satisfies Einstein causality, the time slice axiom and the Noether theorem. The cocycle corresponds to anomalies known in particular from formal perturbation theory and the cocycle relation to the Wess-Zumino consistency relation and the consistency relation in the Batalin-Vilkovisky formalism.

The talk is based on joint work with Detlev Buchholz and with Romeo Brunetti, Michael Dütsch and Kasia Rejzner.

**12:00 – 14:00**

Lunch

**14:00 –**

Free afternoon/Hike

Weather permitting, we will organise a hike to St. Ottilien. We meet at 2 pm in front of the mathematics building Ernst-Zermelo-Straße 1. From there we will hike to Sankt Ottilien which should take about 2 hours. Participants interested in a physically demanding hike can choose to pass over the Rosskopf. At the restaurant of St. Ottilien you have the opportunity to drink and eat something. You may return either by walking back or by calling a taxi cab.

The following link lets you see the hike: <https://www.komoot.de/tour/926228310>



**09:00 – 10:00**

Gerhard Huisken (Tübingen/MFO)

(HS II, Albertstr. 23b, 79104 Freiburg)

*Foliations in asymptotically flat 3-manifolds*

The lecture compares different foliations of the exterior region of an asymptotically flat 3-manifold such as mean curvature flow, inverse mean curvature flow, constant mean curvature foliations and level-sets of suitable solutions to geometric PDEs with a view towards the geometric description of physical concepts such as mass, energy and center of mass.

**10:15 – 11:00**

**Coffee**

**11:00 – 12:00**

Gilles Carron (Nantes)

(HS II, Albertstr. 23b, 79104 Freiburg)

*Euclidean heat kernel rigidity and applications*

I will report on joint work with I. Mondello (Paris 12) and D. Tewodrose (Nantes). With D. Tewodrose, we discover that the Euclidean space can be characterized by its heat kernel. I will explain this result and how it can be used in order to find a crucial  $\varepsilon$ -regularity under very weak condition on the Ricci curvature.

**12:00 – 14:00**

**Lunch**

**14:00 – 15:00**

Rafe Mazzeo (Stanford)

(HS II, Albertstr. 23b, 79104 Freiburg)

*The Extended Bogomolny Equations*

The extended Bogomolny equations on 3-manifolds are the dimensional reduction of the 4-dimensional gauge-theoretic Kapustin-Witten equations theory. The reduction is part of an Atiyah-Floer approach to the construction of solutions of the KW equations with knot singularities. I will report on what is now a near complete understanding of the solutions of these three-dimensional equations, first through joint work with Siqi He, and more recently in the thesis work of Panagiotis Dimakis.

**15:15 – 16:00**

**Coffee**

**16:00 – 17:00**

**Oliver Lindblad Petersen (KTH Stockholm)**

**(HS II, Albertstr. 23b, 79104 Freiburg)**

*Wave equations in subextremal Kerr-de Sitter spacetimes*

In 2013, Vasy proved that solutions to linear wave equations in Kerr-de Sitter spacetimes have asymptotic expansions in quasinormal modes up to an exponentially decaying term, assuming the angular momentum of the black hole satisfies certain bounds. This was the first step towards the proof of non-linear stability for slowly rotating Kerr-de Sitter black holes by Hintz and Vasy in 2018. In this talk, we extend Vasy's result to the full subextremal range of Kerr-de Sitter spacetimes, by removing the restrictions on the angular momentum of the black hole. The proof is based on a new Fredholm setup and a new analysis of the trapping of photons around a Kerr-de Sitter black hole. This is joint work with Andras Vasy.

**17:15 – 18:15**

**Matthias Ludewig (Regensburg)**

**(HS II, Albertstr. 23b, 79104 Freiburg)**

*The stringor bundle of a string manifold*

In an unpublished 2005 draft, Stolz and Teichner proposed that on a string manifold, a certain geometric structure can be defined, which they called the „stringor bundle“. In fact, this geometric structure is a categorified vector bundle, or 2-vector bundle, using the the language we developed together with Kristel and Waldorf. Using recent results from the thesis of Kristel, we a description of this stringor 2-vector bundle. We also explain the relation to the string condition and the spin condition for the loop space.

**18:30 –**

**Conference dinner**

**(Restaurant “Jaipur”, Gerberau 5, 79098 Freiburg)**

Restaurant website: <https://jaipur-freiburg.de>

**09:00 – 10:00**

Dorothee Schüth (HU Berlin)

(HS II, Albertstr. 23b, 79104 Freiburg)

*On the Dirac spectrum of homogeneous 3-spheres*

Starting from Bär's general approach to describing the Dirac spectrum of homogeneous spaces, we show that any two left-invariant metrics on  $S^3 \cong \text{SU}(2)$  which are isospectral for the associated classical Dirac operator  $D$  must be isometric. In the case of left-invariant metrics of positive scalar curvature, we compute and use the smallest eigenvalue of  $D^2$ . This is joint work with Jordi Kling.

**10:15 – 11:00**

**Coffee**

**11:00 – 12:00**

Paolo Piazza (Rome)

(HS II, Albertstr. 23b, 79104 Freiburg)

*Analysis and Geometry of Dirac operators on singular spaces*

In this talk I will present results about Dirac operators on (the regular part of) a smoothly stratified space. I will begin by explaining what can be done under the so-called Witt condition, stressing in particular the case of the signature operator and the spin-Dirac operator. I will also link the analytic results explained for these two particular operators to the geometry of the stratified space. I will then move on to the general case, explaining what can be done beyond the Witt case. The first part of the talk is joint work with various people (Albin, Leichtnam, Mazzeo, Vertman, Botvinnik, Rosenberg) and has appeared in a number of papers; the second part of the talk is ongoing work with Jesse Gell-Redman and Pierre Albin.

**12:00**

**End of conference and lunch**