Structures in Mathematics inspired by Physics

Profile of the Master of Science Program in Mathematics

Institute for Mathematics at Potsdam University

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1 Overview

The fundamental science of Physics has numerous unanswered mathematical questions, in everything from general relativity theory to quantum field theory. These questions have sparked fascinating new developments in many branches of Mathematics, from Analysis and Geometry all the way to Probability Theory. In the specialization Structures in Mathematics inspired by Physics, you will obtain the necessary basic knowledge and apply them to today's hottest research fields.

The fundamental concepts are taught in courses offered each year in the branches of Differential Geometry, Functional Analysis, Partial Differential Equations and Probability Theory. In advanced courses at the end of the program, you can further specialize in two or more of these branches. Lectures and seminars on current research topics, some held jointly with the Albert Einstein Institute for Gravitational Physics, offer a glimpse into today's modern research.

Successful completion of a Master's program under this profile opens doors for the student in a wide variety of professional careers. Beyond these, students will be prepared to pursue a PhD.

2 Structure of the Profile

The Institute for Mathematics offers the following courses in this profile:

- Basic courses (9 LP each):
  - Differential Geometry 1 (summer term)
  - Functional Analysis 1 (winter term)
  - Partial Differential Equations 1 (winter term)
  - Stochastic Processes (summer term)
- Advanced courses (9 LP each):
  - Differential Geometry 2 (winter term)
  - Functional Analysis 2 (summer term)
  - Partial Differential Equations 2 (summer term)
  - Stochastic Analysis (winter term)
- Specialized courses consisting of lectures or seminars (6-9 LP) on selected topics of the following:
  - Analysis
  - Differential Geometry
  - Geometric Analysis
  - Probability Theory

For completion of a Master's program under this profile the students successfully complete:

1. At least six of the basic and advanced courses from the list above. If some of these courses were part of the Bachelor degree they have to be replaced by other courses.
2. Courses from a minor subject with an amount of about 18 LP. It is recommended to attend courses in physics or specialized courses of this profile.
3. Moreover, the Master's program comprises two seminars (6 LP each), a scientific project (6 LP) and the Master's thesis (30 LP). It is recommended to choose the seminars and the scientific project from the specialized courses of this profile. Ideally they should be a preparation for the Master's thesis.
4. The exact requirements can be found in the study regulations for Master of Mathematics.

3 Short Description of the Courses

Differential Geometry 1

- Manifolds and vector bundles
- Metrics, connections, geodesics, curvature, Jacobi fields
- Submanifolds
- Semi-Riemannian geometry
- Applications to theoretical physics

Differential Geometry 2

The contents of this course vary between the following topics:

- Riemannian geometry
- Lorentzian geometry
- Symplectic geometry
- Spin geometry

Functional Analysis 1

- Fundamental results on Banach and Hilbert spaces (orthogonal bases and projections; Hahn-Banach, Baire Category Theorem and their consequences)
- Distributions and Sobolev spaces, Fourier transform
- Compact operators, Fredholm operators

Functional Analysis 2

- Spectral theorem for bounded and unbounded self-adjoint operators
- Stone's theorem
- Quadratic forms and applications to unbounded self-adjoint operators
- Applications: electric and magnetic fields, Feynman-Kac formula

Partial Differential Equations 1

- Fundamental examples: wave equation, heat equation, Poisson equation
- The Perron method
- Linear elliptic equations in Sobolev spaces

Partial Differential Equations 2

The contents of this course vary between the following topics:

- Methods in the calculus of variations
- Linear and non-linear wave equations
- Microlocal Analysis

Stochastic Processes

- Markov chains
- Convergence to equilibrium, reversibility
- Examples: random walk, birth and death process, Poisson process

Stochastic Analysis

- Brownian motion: construction and properties
- Stochastic integration and Itô differential calculus
- Stochastic differential equations: theory and examples (Langevin equation, geometric Brownian motion)
- Martingale theory
- Time changed Brownian motion

Topics in Analysis

Courses on the following topics are offered:

- Non-linear analysis on Banach spaces
- Real analysis
- Rectifiability, currents und variolds
- Regularisation in mathematics and physics
- Traces and Determinants in mathematics and physics, anomalies
- Renormalization on cones
- Dynamical systems
- Analytic mechanics and symplectic geometry
- Scattering theory

Topics in Differential Geometry

Courses on the following topics are offered:

- Spin geometry
- Spectral geometry
- Introduction to general relativity
- Gauge theory
- Global analysis

Topics in Geometric Analysis

Courses on the following topics are offered:

- Geometric variational problems: minimal surfaces, Willmore surfaces, harmonic maps
- Geometric evolution equations: mean curvature flow, Ricci flow

Topics in Probability Theory

Courses on the following topics are offered:

- Lévy processes
- Limit theorems in probability theory
- Stochastic modelling
- Branching processes
- Statistical mechanics

4 Course of Studies Example

Start in winter term

1. Semester (Winter)	2. Semester (Summer)	3. Semester (Winter)	4. Semester (Summer)
Functional Analysis 1 (9 LP)	Differential Geometry 1 (9 LP)	Advanced Course 2 (9 LP)	Master's thesis (30 LP)
Partial Differential Equations 1 (9 LP)	Stochastic Processes (9 LP)	Seminar 1 (6 LP)	
Scientific Project (6 LP)	Advanced Course 1 (9 LP)	Seminar 2 (6 LP)	
Minor subject (6 LP)	Minor subject (6 LP)	Minor subject (6 LP)	
30 LP	33 LP	27 LP	30 LP

Start in summer term

1. Semester (Summer)	2. Semester (Winter)	3. Semester (Summer)	4. Semester (Winter)
Differential Geometry 1 (9 LP)	Functional Analysis 1 (9 LP)	Advanced Course 1 (9 LP)	Master's thesis (30 LP)
Stochastic Processes (9 LP)	Partial Differential Equations 1 (9 LP)	Advanced Course 2 (9 LP)	
Seminar 1 (6 LP)	Seminar 2 (6 LP)	Scientific Project (6 LP)	
Minor subject (6 LP)	Minor subject (6 LP)	Minor subject (6 LP)	
30 LP	30 LP	30 LP	30 LP

5 Contact

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