Rotating black holes in 5D Einstein-Maxwell-Chern-Simons theory with negative cosmological constant

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RESEARCH TRAINING GROUP
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Rotating black holes in 5D Einstein-Maxwell-Chern-Simons theory with negative cosmological constant

1. Introduction
2. General properties of EMCS-AdS black holes
3. Near-horizon formalism
4. Numerical and analytical results

1. Introduction

|| 1. Introduction ||

Black holes in D=5 dimensions in Einstein-Maxwell-Chern-Simons theory with negative cosmological constant

Asymptotically anti-de-Sitter space-times:

Interesting in the context of the AdS/CFT correspondence

Gravitating fields propagating in an AdS space-time

Fields propagating in a conformal field theory

Black holes in higher dimensions have some special properties:

- Topologies of stationary black holes can be non-spherical
  For example black ring solution (Emparan 2002)
- More than one independent plane of rotation
  In D dimensions there are N = [(D-1)/2] planes of rotation

N independent angular momenta

|| 1. Introduction ||

We are interested in the higher dimensional generalization of the Kerr-Newman black holes:

Axisymmetric and stationary,
Spherical topology of the horizon,
Electrically charged

All angular momenta of the same magnitude:

$$|\mathbf{J}|=|\mathbf{J}_1|=|\mathbf{J}_2|=\ldots=|\mathbf{J}_N|$$

We have enhanced *U(N)* symmetry

Not even with these constraints uniqueness is granted

We use numerical methods to obtain
global solutions with these properties.

We also make use of the near-horizon formalism.

2. General properties of EMCS-AdS black holes

|| 2. General Properties ||

$$I = \frac{1}{16\pi G_5}\int d^5x\left[\sqrt{-g}(R - F^2 - 2\Lambda) - \frac{2\lambda}{3\sqrt{3}}\varepsilon^{\mu\nu\alpha\beta\gamma}A_\mu F_{\nu\alpha}F_{\beta\gamma}\right]$$

Einstein-Maxwell-Chern-Simons theory in 5 dimensions

R = curvature scalar

Gravity coupled to a U(1) electro-magnetic potential A_μ
with a Chern-Simons term (D=5) and a cosmological constant

F = field strength tensor

Λ = cosmological constant

λ = Chern-Simons coupling parameter

|| 2. General Properties ||

$$I = \frac{1}{16\pi G_5}\int d^5x\left[\sqrt{-g}(R - F^2 - 2\Lambda) - \frac{2\lambda}{3\sqrt{3}}\varepsilon^{\mu\nu\alpha\beta\gamma}A_\mu F_{\nu\alpha}F_{\beta\gamma}\right]$$

Einstein-Maxwell-Chern-Simons theory in 5 dimensions

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 2\left(F_{\mu\rho}F^{\rho}{}_{\nu} - \frac{1}{4}F^2\right)$$

Einstein equations

$$G_5 = 1$$

$$\nabla_\nu F^{\mu\nu} + \frac{\lambda}{2\sqrt{3}}\varepsilon^{\mu\nu\alpha\beta\gamma}F_{\nu\alpha}F_{\beta\gamma} = 0$$

Maxwell equations

|| 2. General Properties ||

Second-order differential equations, non-linear.

We are interested in a special subset of the solutions:

- Axially symmetric and stationary: N+1 Killing vector fields

$\xi = \partial_t$ → Asymptotically time-like vector of unitary norm

$\eta_{(k)} = \partial_{\varphi_k}$ → N asymptotically space-like vector fields (cyclic)

$$\pounds_\xi g = \pounds_{\eta_{(k)}} g = 0$$

We can associate an independent angular momentum $J_{(k)}$ with each vector field (independent planes of rotation)

$U(1)^N$ symmetry

|| 2. General Properties ||

Ansatz constraints:

1. Axially symmetric and stationary: $U(1)^N$ symmetry

2. All angular momenta of equal magnitude: enhanced $U(N)$ symmetry

$$| J_{(1)} | = | J_{(2)} | = \ldots = | J_{(N)} | = J$$

3. Event horizon with spherical topology

4. Asymptotically AdS

|| 2. General Properties ||

Ansatz for the metric (5D):

$$ds^2 = -b(r)dt^2 + \frac{1}{u(r)}dr^2 + g(r)d\theta^2 + p(r)\sin^2\theta\left(d\varphi_1 - \frac{\omega(r)}{r}dt\right)^2$$

$$+p(r)\cos^2\theta\left(d\varphi_2 - \frac{\omega(r)}{r}dt\right)^2 + (g(r) - p(r))\sin^2\theta\cos^2\theta(d\varphi_1 - d\varphi_2)^2$$

$$\theta \in [0, \pi/2],\ \varphi_1 \in [0, 2\pi] \text{ and } \varphi_2 \in [0, 2\pi]$$

Lewis-Papapetrou coordinates. The radial coordinate **r** is isotropic.

Ansatz for the gauge field:

$$A_\mu dx^\mu = a_0(r)dt + a_\varphi(r)(\sin^2\theta d\varphi_1 + \cos^2\theta d\varphi_2)$$

System of second order ordinary differential equations + constraints

|| 2. General Properties ||

A convenient way to parametrize the functions is

$$\Lambda = -\frac{6}{L^2}$$

$$u(r) = \frac{f(r)}{m(r)}\left(\frac{r^2}{L^2}+1\right) \qquad g(r) = \frac{m(r)}{f(r)}r^2$$

$$b(r) = f(r)\left(\frac{r^2}{L^2}+1\right) \qquad p(r) = \frac{n(r)}{f(r)}r^2$$

Asymptotical behaviour at infinity:

$r \to \infty$:

$$f(r) = 1 + \frac{\alpha}{r^4} + o\left(\frac{1}{r^6}\right) \qquad \omega(r) = \frac{\hat{J}}{r^3} + o\left(\frac{1}{r^5}\right)$$

$$m(r) = 1 + \frac{\beta}{r^4} + o\left(\frac{1}{r^6}\right) \qquad a_0(r) = \frac{-q}{r^2} + o\left(\frac{1}{r^6}\right)$$

$$n(r) = 1 + \frac{3(\alpha-\beta)}{r^4} + o\left(\frac{1}{r^6}\right) \qquad a_\varphi(r) = \frac{\hat{\mu}}{r^2} + o\left(\frac{1}{r^6}\right)$$

|| 2. General Properties ||

Event horizon → Killing horizon:

Axisymmetric and stationary black holes
Rigidity theorem in higher dimensions

S. Hollands, A. Ishibashi, and R. Wald, Communications in Mathematical Physics 271 (2007) 699

S. Hollands and A. Ishibashi, Communications in Mathematical Physics 291 (2009) 443

S. Hollands and A. Ishibashi, Classical and Quantum Gravity 29 (2012) 1630010

Killing vector field associated to the horizon
Light-like vector field at the horizon
The horizon is a light-like hypersurface.

$$\chi = \xi + \Omega \sum_{k=1}^{N} \varepsilon_k \eta_{(k)}$$

Ω is the angular velocity of the event horizon
as seen from an inertial frame at infinity
(equal magnitude angular momenta)
Rigid rotation of the event horizon

|| 2. General Properties ||

System of second-order differential equations for **(f(r), m(r), n(r), ω(r), $\mathbf{a_\varphi}$(r))**

Conservation of the electric charge: **$\mathbf{a_0}$(r)** decoples from the system

Q is introduced into the differential equations

Boundary conditions:

Regularity at the Killing horizon:

$$f|_{r=r_{\rm H}} = m|_{r=r_{\rm H}} = n|_{r=r_{\rm H}} = 0$$

$$\omega|_{r=r_{\rm H}} = r_{\rm H}\Omega$$

Asymptotically AdS:

$$f|_{r=\infty} = m|_{r=\infty} = n|_{r=\infty} = 1$$

$$\omega|_{r=\infty} = 0$$

Gauge fixing: $$a_0|_{r=\infty} = a_\varphi|_{r=\infty} = 0$$

|| 2. General Properties ||

Global Charges:

Mass

$$M = -\frac{\pi}{8}\frac{\beta - 3\alpha}{L^2}$$

(Ashtekar-Magnon-Das conformal mass)

Angular Momenum

$$J_{(k)} = \int_{S^3_\infty} \beta_{(k)}$$

$$\beta_{(k)\mu_1\mu_2\mu_3} \equiv \epsilon_{\mu_1\mu_2\mu_3\rho\sigma}\nabla^\rho \eta^\sigma_{(k)}$$

$$|J_{(k)}| = J$$

Electric charge

$$Q = -\frac{1}{2}\int_{S^3_\infty} \tilde{F}$$

$$\tilde{F}_{\mu_1\mu_2\mu_3} \equiv \epsilon_{\mu_1\mu_2\mu_3\rho\sigma}F^{\rho\sigma}$$

|| 2. General Properties ||

Horizon Charges:

Area

$$A_{\rm H} = \int_{\mathcal{H}} \sqrt{|g^{(3)}|} = 2\pi^2 r_{\rm H}^3 \lim_{r\to r_{\rm H}} \sqrt{\frac{m^2 n}{f^3}}$$

Entropy

$S = 4\pi A_H$

Horizon Mass

$$M_{\rm H} = -\frac{3}{2}\int_{\mathcal{H}} \alpha = \lim_{r\to r_{\rm H}} 2\pi^2 r^3 \sqrt{\frac{mn}{f^3}} \left[\frac{n\omega}{f}\left(\frac{\omega}{r} - \omega'\right) + f'\left(1 + \frac{r^2}{L^2}\right) + \frac{2rf}{L^2}\right.$$

Horizon Angular Momenta

$$J_{{\rm H}(k)} = \int_{\mathcal{H}} \beta_{(k)} = \lim_{r\to r_{\rm H}} \pi^2 r^3 \sqrt{\frac{mn^3}{f^5}} \left[\omega - r\omega'\right]$$

|| 2. General Properties ||

Extremal black holes.

Several definitions:

1. Minimal mass for fixed angular momenta and electric charge.

Hence extremal black holes are interesting to characterize the domain of existence of the solutions.

This definition could be inappropiate if the is not uniqueness of the solutions with the global charges

2. Vanishing surface gravity (temperature).

There are examples of black holes with non-vanishing surface gravity
G. Gibbons and K. ichi Maeda, Nuclear Physics B 298 (1988) 741

3. Degeneracy of the roots of the g^{rr} component.

Some times the ergosphere coincides with the horizon: Ergo-free black holes

|| 2. General Properties ||

Extremal black holes present non integer exponents in their horizon expansion.

- ✓ EM case
- ✓ EMCS case

Special parametrization of the functions in order to numerically solve the problem.

$$f = f_4 r^4 + f_\alpha r^\alpha + o(r^6)$$
$$m = m_2 r^2 + m_\beta r^\beta + o(r^4)$$
$$n = n_2 r^2 + n_\gamma r^\gamma + o(r^4)$$
$$\omega = \omega_1 r + \omega_2 r^2 + o(r^3)$$
$$a_0 = a_{0,0} + a_{0,\lambda} r^\lambda + o(r^2)$$
$$a_\phi = a_{\phi,0} + a_{\phi,\mu} r^\mu + o(r^2)$$

[redacted]

$4 < \alpha < 6$	$2 < \beta < 4$	$2 < \gamma < 4$
$0 < \lambda < 2$	$0 < \mu < 2$	[redacted]

|| 2. General Properties ||

Extremal black holes with non-vanishing cosmological constant

- ✓ EM-AdS case
- ✓ EMCS-AdS case

We also use a special parametrization of the functions in order to numerically solve the problem.

$$n(x) = n_0 x^\alpha + o\left(x^{(\alpha+1)}\right)$$
$$m(x) = m_0 x^\beta + o\left(x^{(\beta+1)}\right)$$
$$f(x) = f_0 x^\gamma + o\left(x^{(\gamma+1)}\right)$$
$$\omega(x) = \omega_1 x + o\left(x^2\right)$$
$$a_\varphi(x) = a_{(\varphi,0)} + a_{(\varphi,1)} x^\delta + o\left(x^{(\delta+1)}\right)$$

$$\alpha > 2,\ \beta > 2,\ \gamma > 4 \text{ and } \delta > 2$$

λ=1

Analytical solution:

Rotating Black Holes in Minimal Five-Dimensional Gauged Supergravity

Chong, Cvetic, Lu, Pope (2005)

$$\alpha = \beta = 4\frac{\sqrt{3r_+^2 + 2a^2 + L^2}}{\sqrt{L^2 - a^2}} - 2$$
$$\gamma = 4\frac{\sqrt{3r_+^2 + 2a^2 + L^2}}{\sqrt{L^2 - a^2}}$$

3. Near-horizon formalism

|| 3. Near Horizon Formalism ||

The space-time outside the event horizon of extremal black holes can be divided in two different regions:

- Near-horizon geometry
- Bulk geometry

Extracting the NHG from a known analytical solution
by a coordinate transformation:

1. Move to a frame comoving with the event horizon
2. Center the radial coordinate on the event horizon
3. Scale parameter Λ in the new radial and temporal coordinates.
4. Series expansion for small Λ

$$r = (\Lambda y + a)$$

$$dt = \frac{a_0}{\Lambda} dT + (a_2/r^2 + a_1/r)dr$$

First term is scale independent: near-horizon geometry

|| 3. Near Horizon Formalism ||

Properties of the near-horizon geometry of extremal black holes.

H. K. Kunduri and J. Lucietti, Living Reviews in Relativity 16 (2013)

- Extremal black holes with spherical topology: near-horizon geometry is the product of two independent spaces.

$$AdS_2 \times S^{D-2}$$

Isometries: $SO(2,1) \times SO(D-1)$ static case (sphere)

$SO(2,1) \times U(1)^N$ rotation (squashed sphere)

This factorization is obtained for all the known examples of topologically spherical black holes

|| 3. Near Horizon Formalism ||

Hence we can assume such factorization in our black holes (extremal case)

Metric:

$$ds^2 = v_1(dr^2/r^2 - r^2dt^2) + v_2[4d\theta^2 + \sin^2 2\theta(d\phi_2 - d\phi_1)^2]$$
$$+v_2\eta[d\phi_1 + d\phi_2 + \cos^2 2\theta(d\phi_2 - d\phi_1) - \alpha r dt]^2$$

Gauge potential:

$$A = -(\rho + p\alpha)rdt + 2p(\sin^2\theta d\phi_1 + \cos^2\theta d\phi_2)$$

- Field equations + Ansatz: algebraic relations for the Ansatz parameters
- Alternatively: Extremal of entropy functional

- Global charges can be calculated: **(J, Q)**
- Horizon charges: area, horizon angular momentum

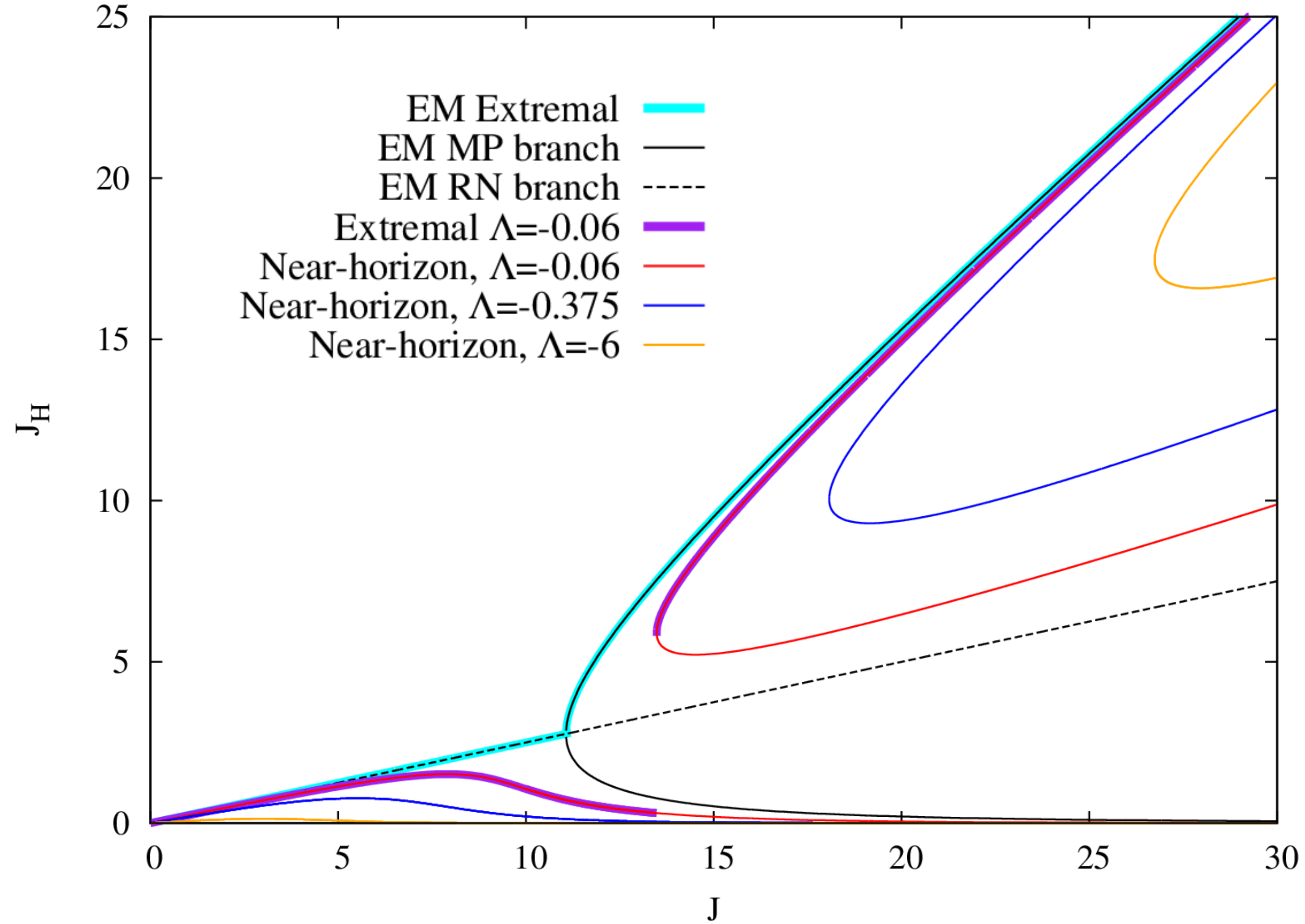
- Parameters related to the asymptotical structure of the global solution cannot be calculated: Mass, angular velocity

4. Numerical and analytical results

Global solutions and branch structure for the pure Einstein-Maxwell case (λ=0)
Flat vs AdS

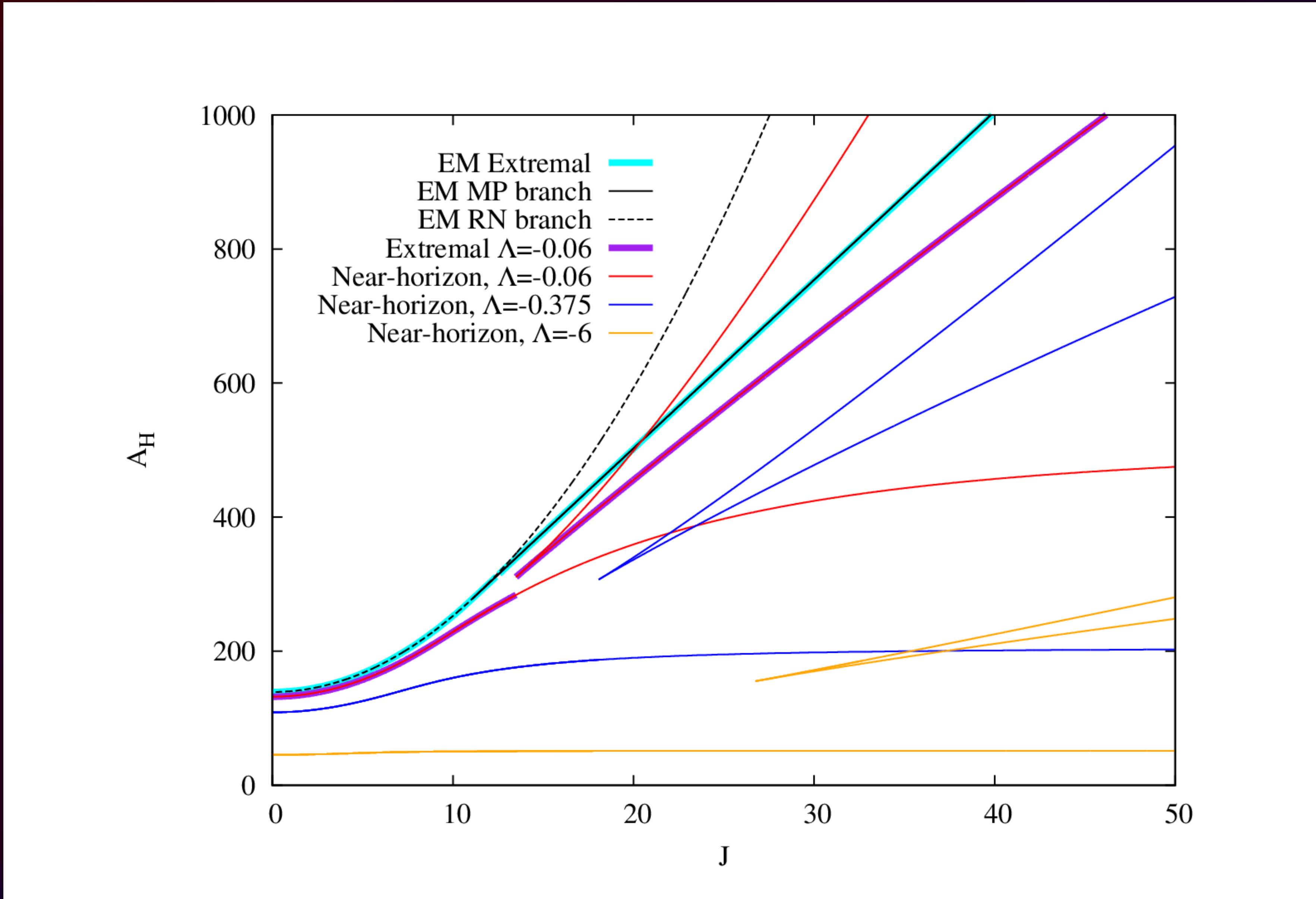
|| 4. Numerical Results ||

Black holes with Q=1 (extremal and NH solutions)



|| 4. Numerical Results ||

Black holes with Q=1 (extremal and NH solutions)

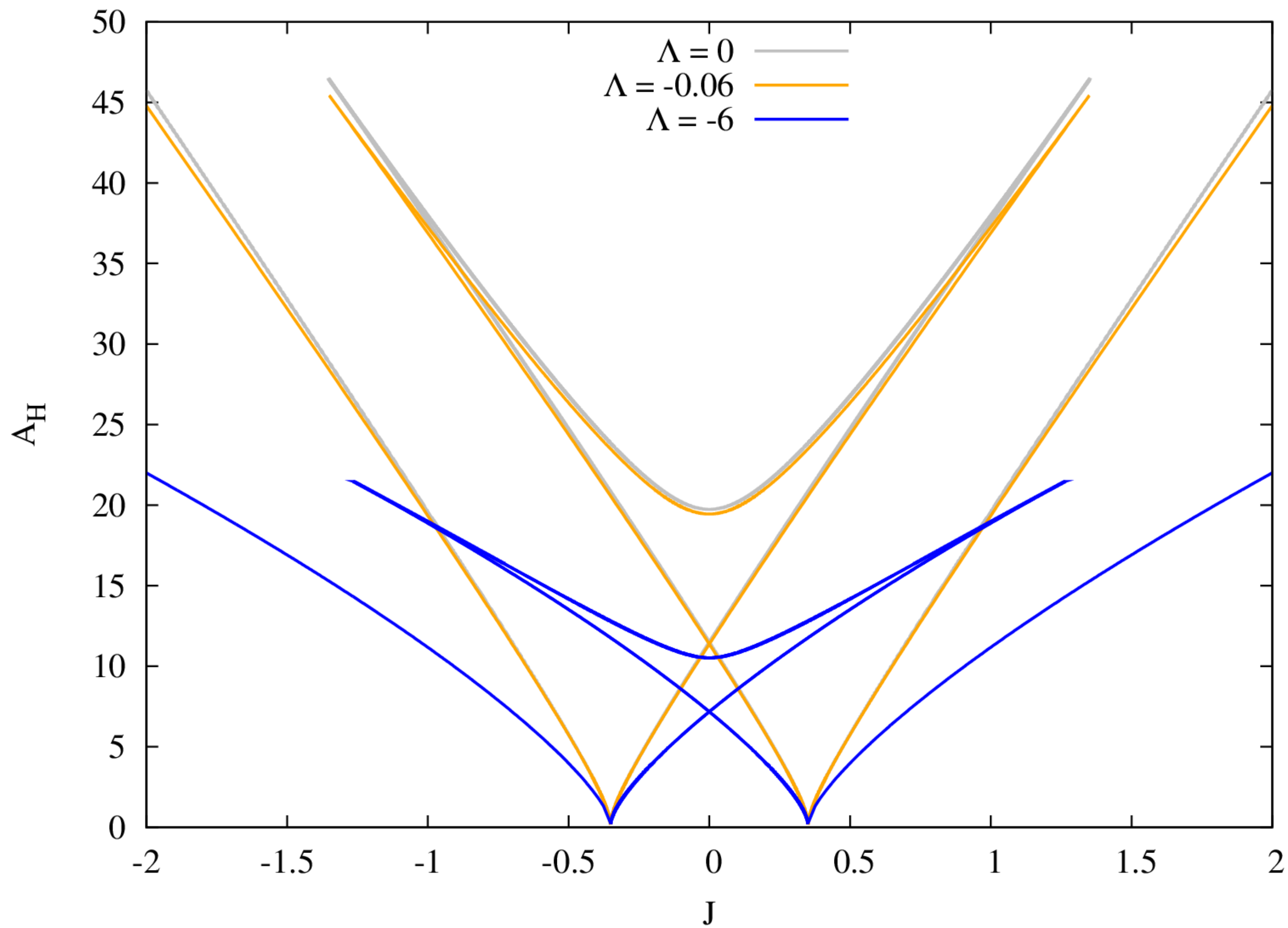


4. Numerical and analytical results

Global solutions and branch structure
$\lambda>2$

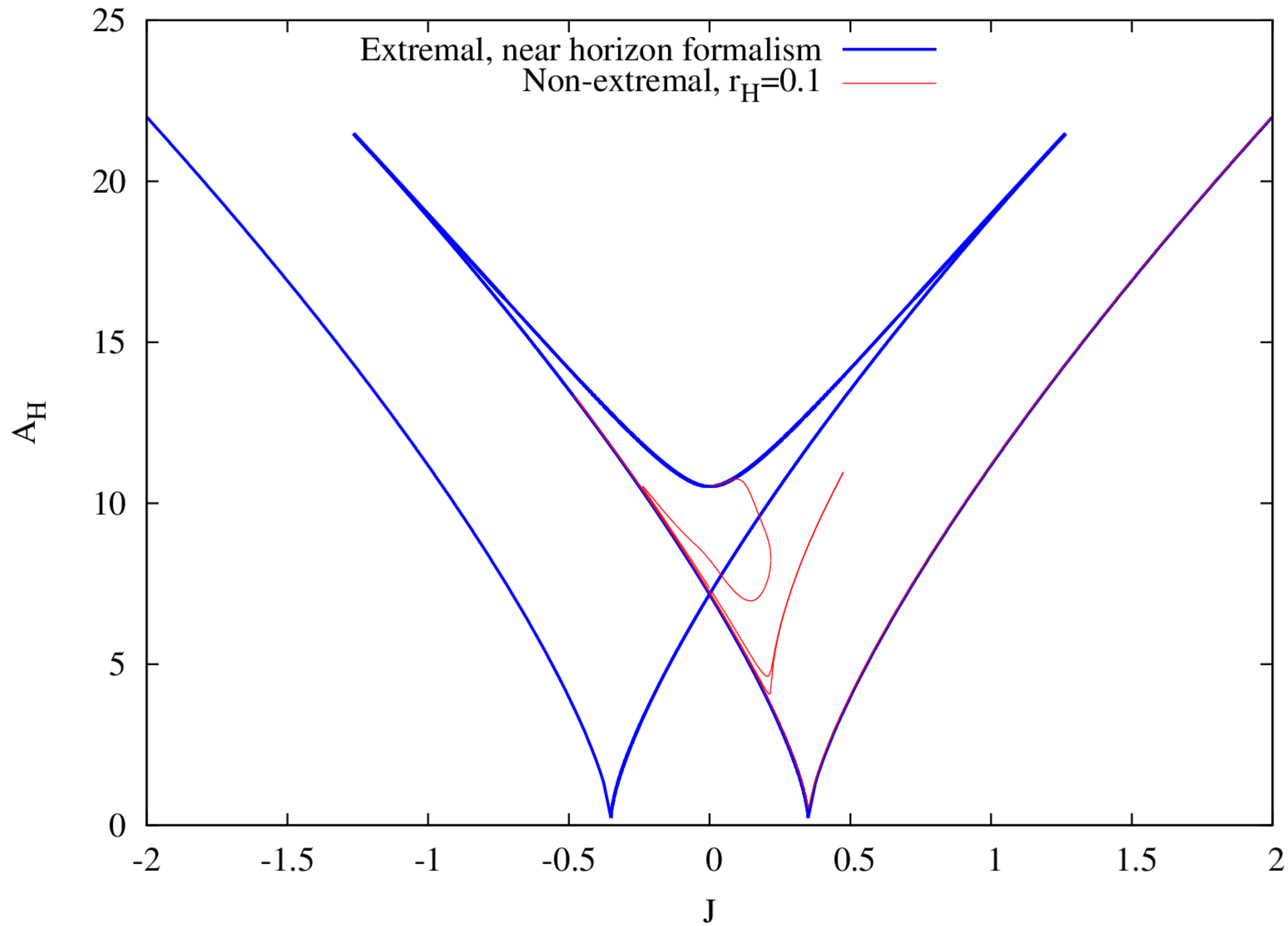
|| 4. Numerical Results ||

Near-horizon solutions for Q=2.720699, λ=5



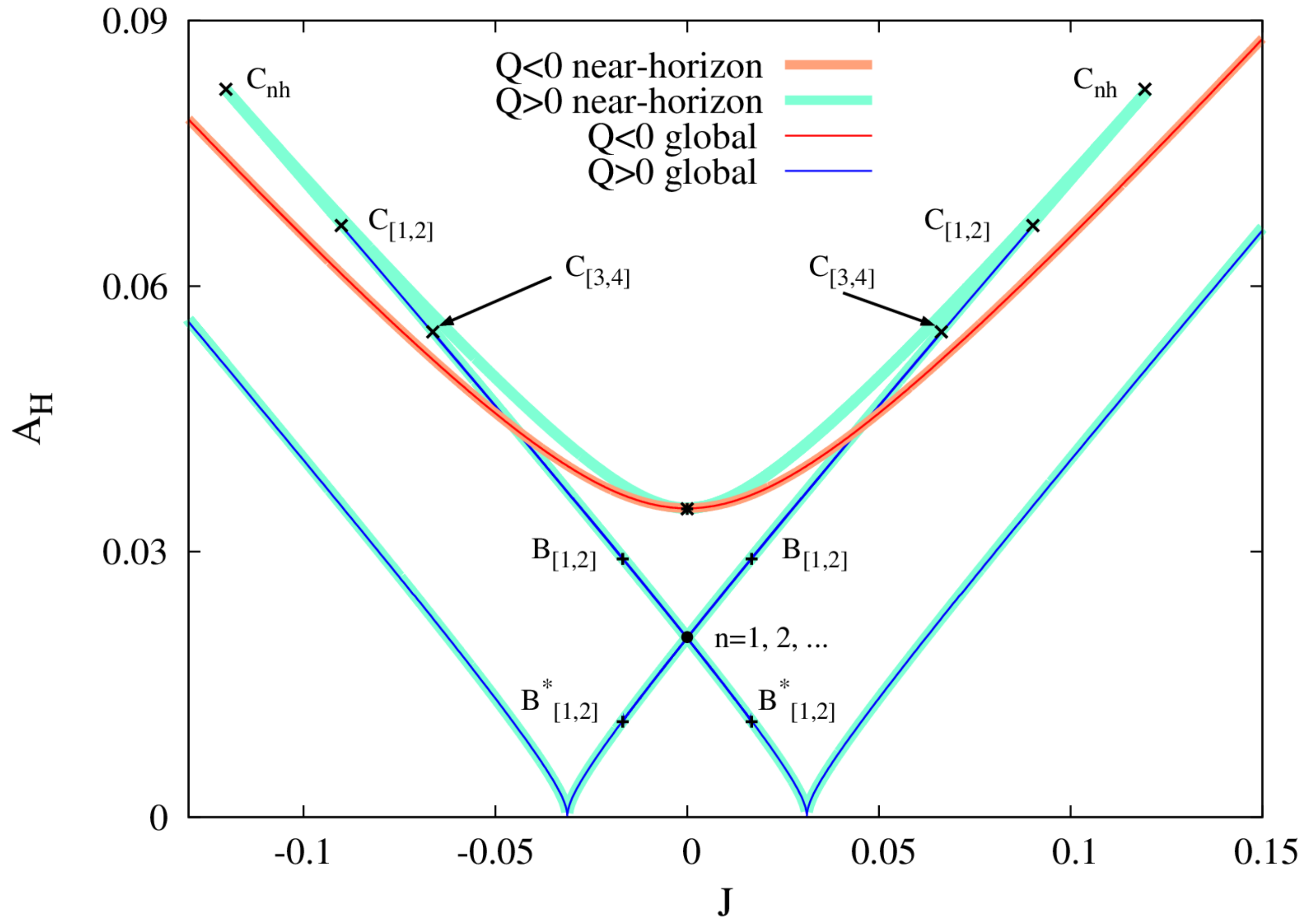
|| 4. Numerical Results ||

Near-horizon and global solutions for Q=2.720699, λ=5, Λ=-6



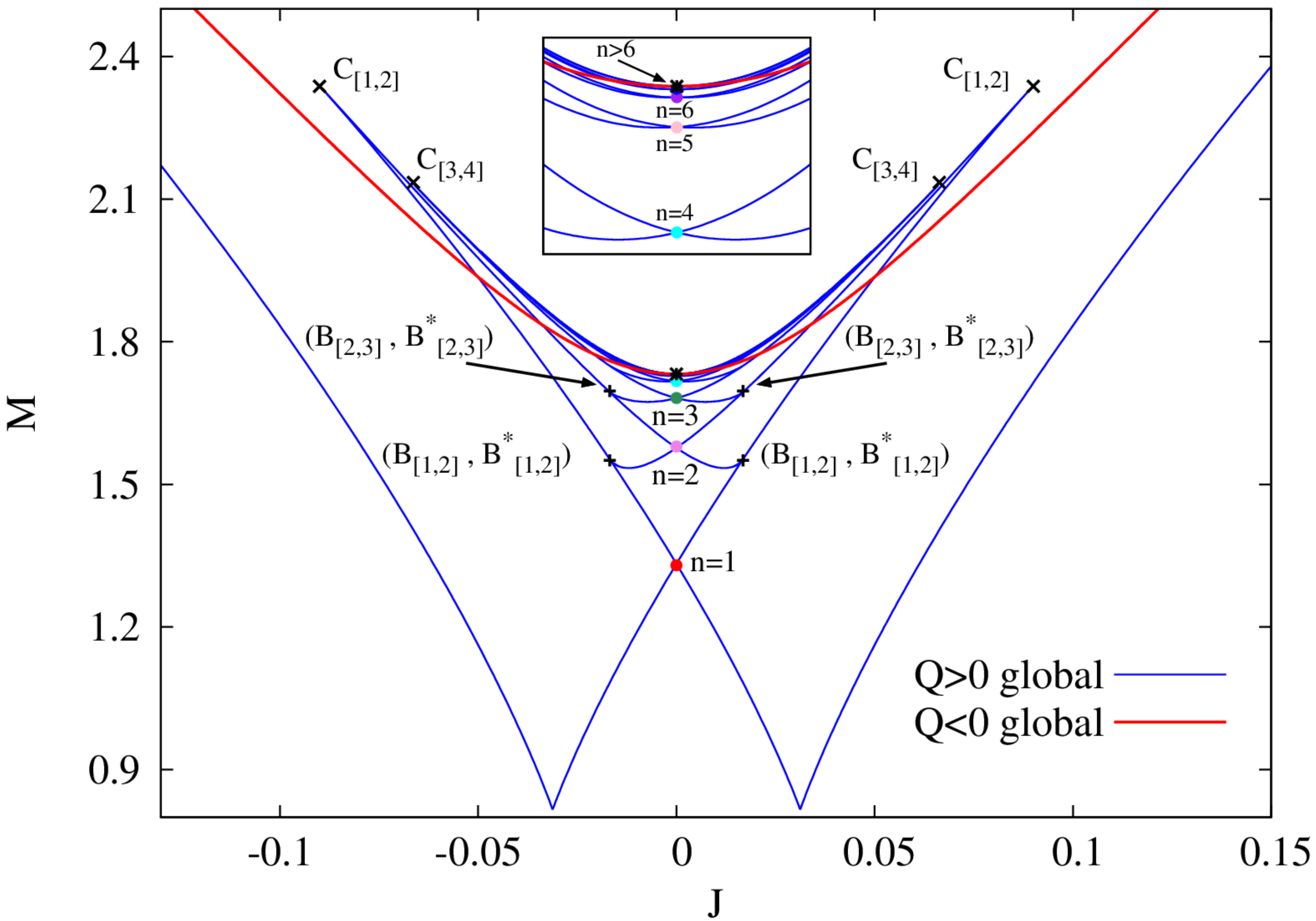
|| 4. Numerical Results ||

Global and NH solutions, Q=1/2π, λ=5



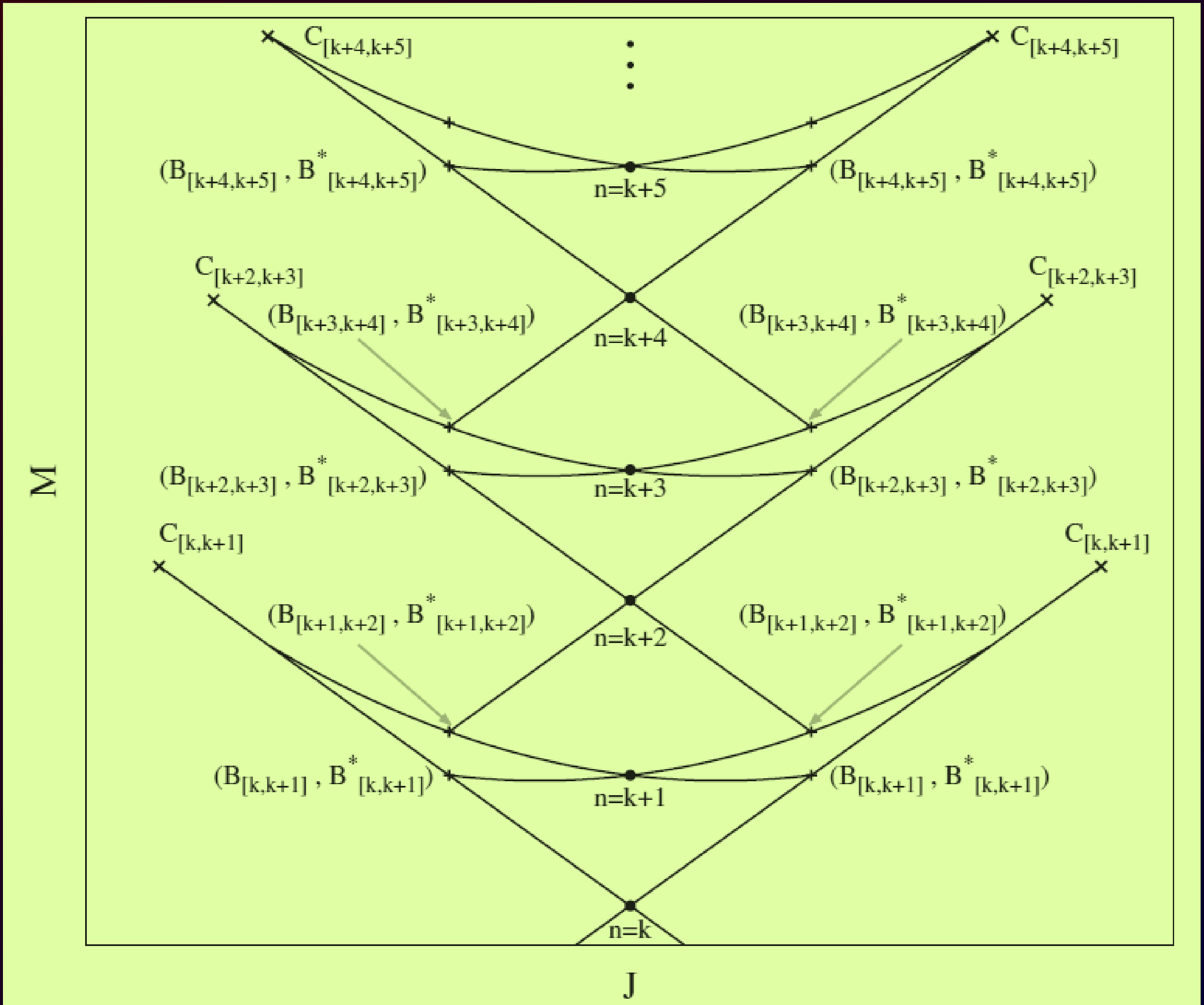
|| 4. Numerical Results ||

Global solutions, Q=1/2π, λ=5

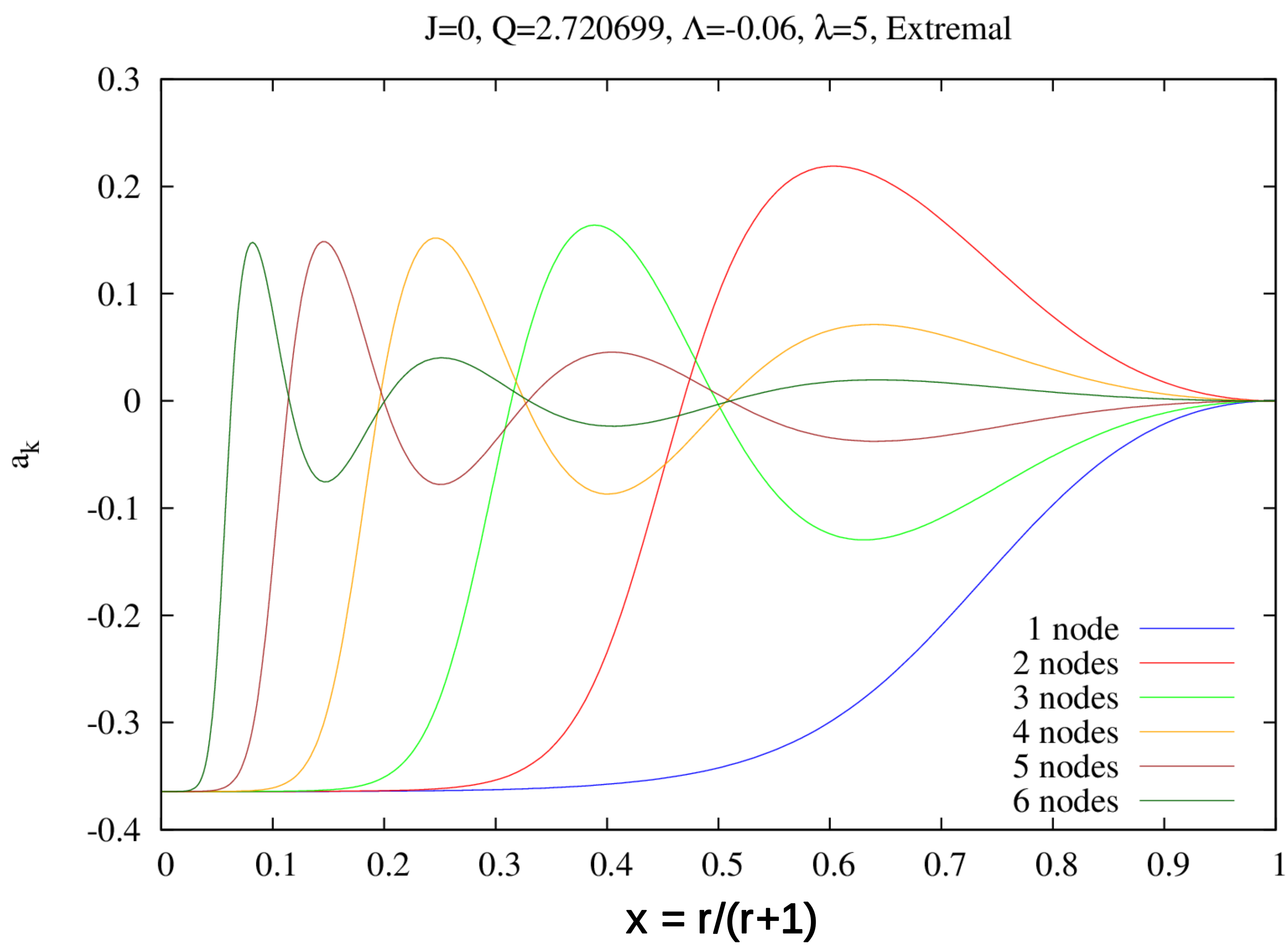


|| 4. Numerical Results ||

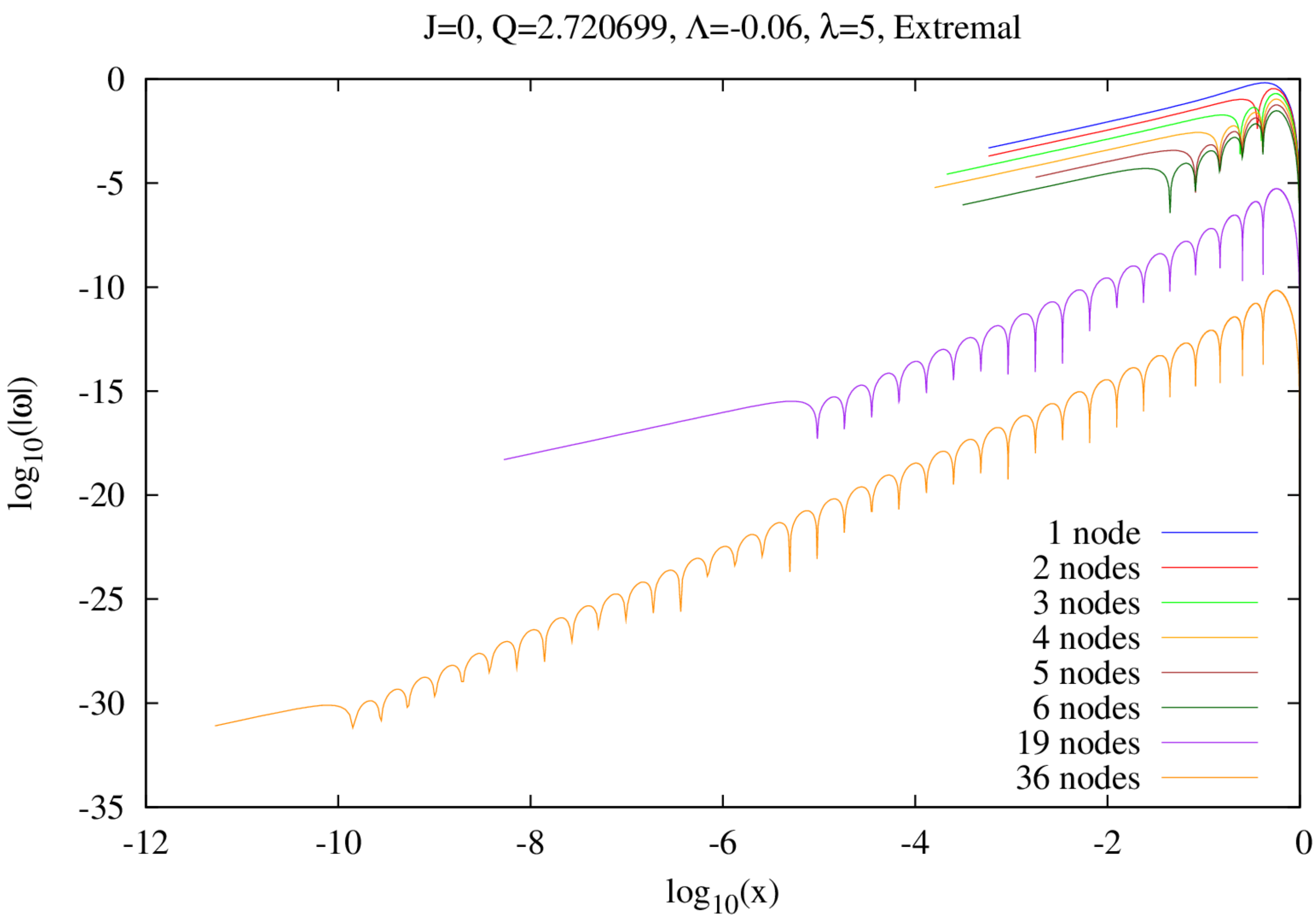
Branch structure scheme:



|| 4. Numerical Results ||



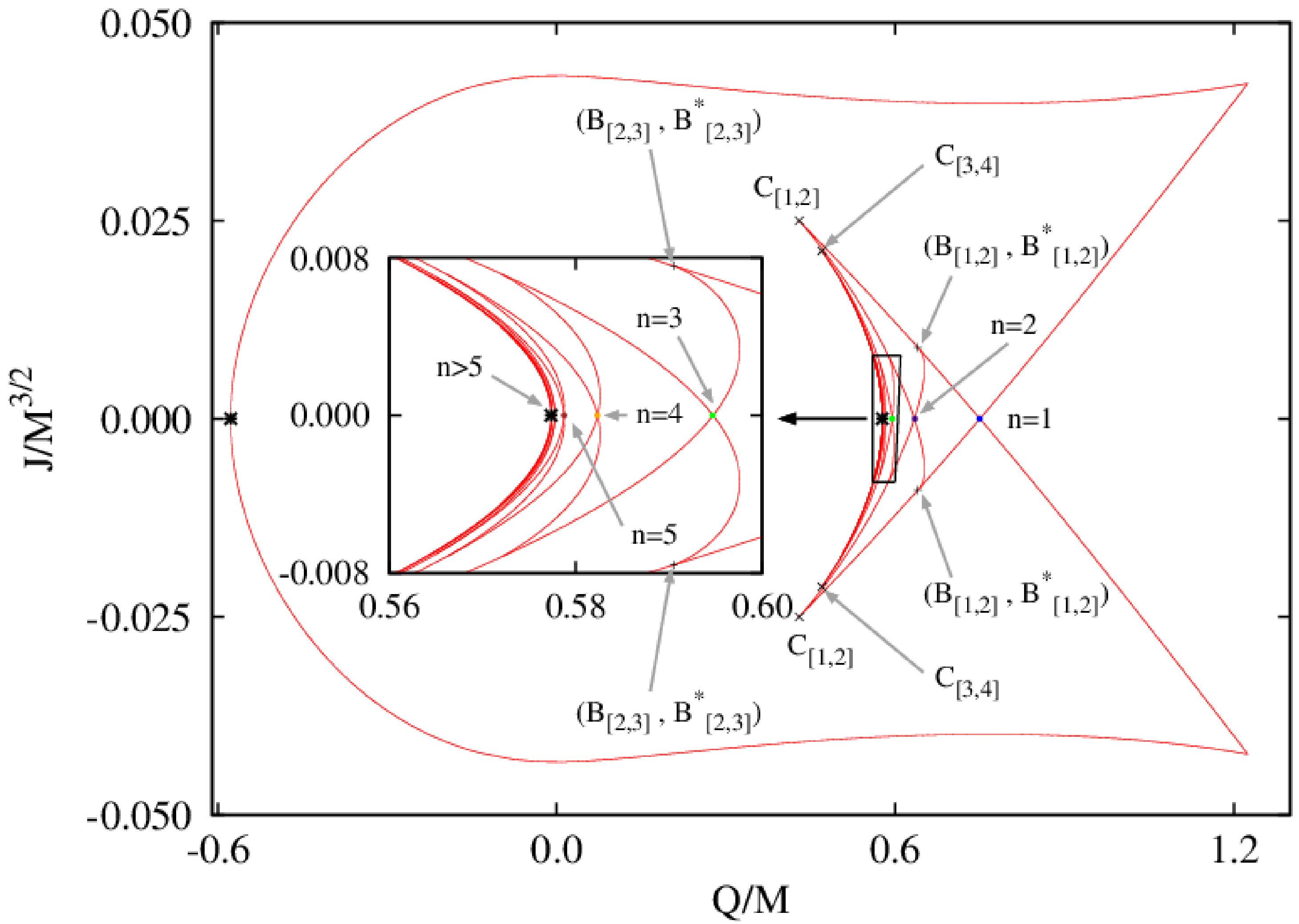
|| 4. Numerical Results ||



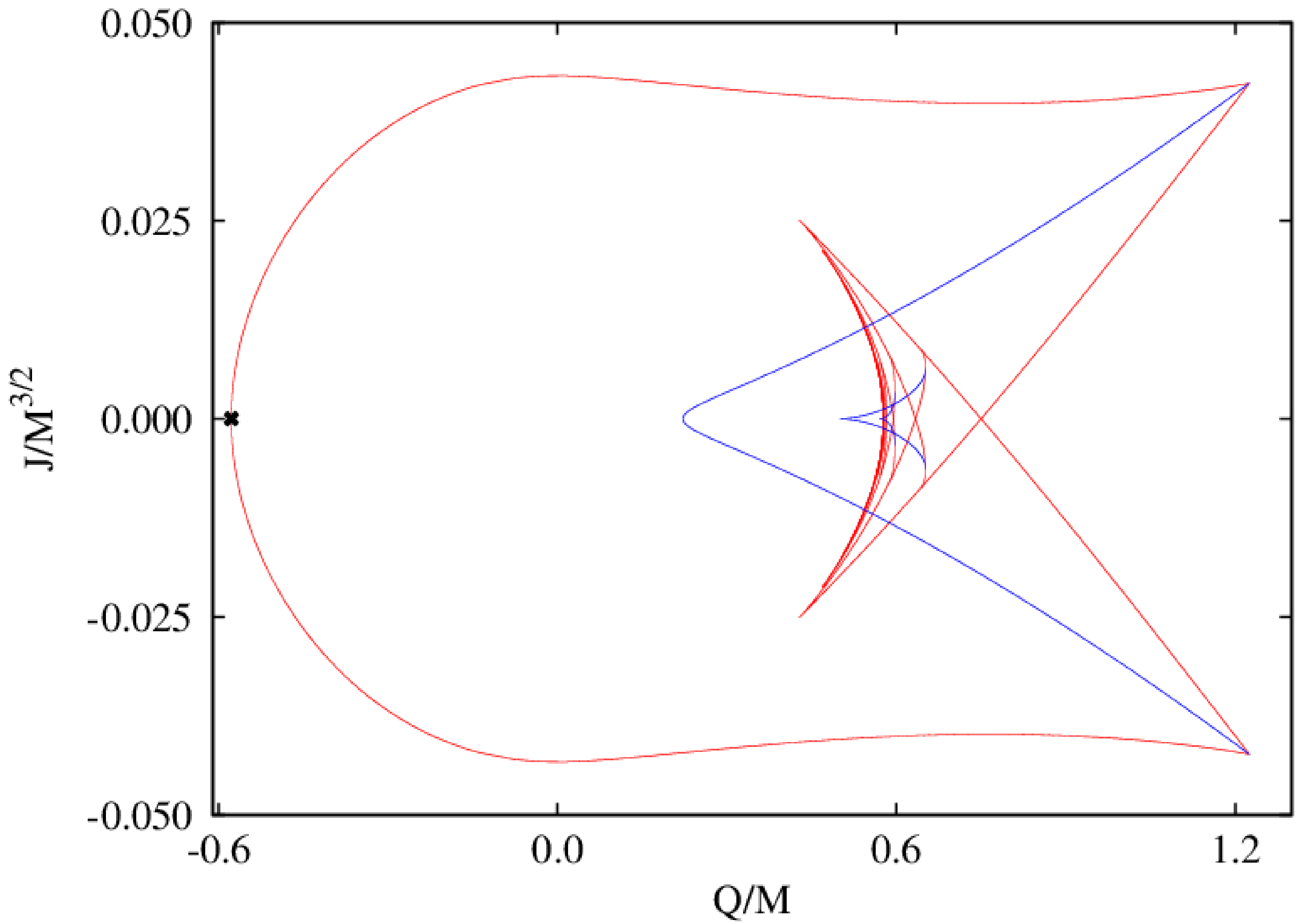
4. Numerical and analytical results

Domain of existence of EMCS black holes with $\lambda>2$

Domain of existence: Extremal solutions



Extremal vs $\Omega_H=0$



Thank you for your attention!

Jose Luis Blazquez-Salcedo, Jutta Kunz, Francisco Navarro Lerida, Eugen Radu, Sequences of Extremal Radially Excited Rotating Black Holes, Physical Review Letters **112** (2014) 011101