Variational formulations of General Relativity

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Mathematical General Relativity

Introduction

Variational principles in General Relativity:

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- metric picture:

$$\mathcal{L}_g = \mathcal{L}_g\left(g_{\mu\nu},\, g_{\mu\nu,\alpha}\,, g_{\mu\nu,\alpha\beta},\, \phi,\, \phi_{,\nu}\right) = \mathcal{L}_H(\mathring{R}) + \mathcal{L}_{matt}\,,$$

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Idea:

All of this formulations are equivalent *on shell*.

Metric Hilbert Lagrangian:

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$$\mathcal{L}_H(\overset{\circ}{R}) = \frac{\sqrt{|\det g|}}{16\pi} g^{\mu\nu} \overset{\circ}{R}_{\mu\nu} := \pi^{\mu\nu} \overset{\circ}{R}_{\mu\nu} ,$$

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$$\begin{aligned} \delta\mathcal{L}_{matt} &= -\frac{1}{16\pi} \overset{\circ}{\mathcal{G}}^{\mu\nu} \delta g_{\mu\nu} + \partial_\nu \left(\pi_\kappa{}^{\lambda\mu\nu} \delta \overset{\circ}{\Gamma}{}^\kappa{}_{\lambda\mu} \right) = \\ &= \overset{\circ}{R}_{\mu\nu} \delta\pi^{\mu\nu} + \partial_\nu \left(\pi_\kappa{}^{\lambda\mu\nu} \delta \overset{\circ}{\Gamma}{}^\kappa{}_{\lambda\mu} \right) , \end{aligned}$$

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\frac{\partial \mathcal{L}_{matt}}{\partial \phi} &= -\sqrt{|\det g|} \, m^2 \, \phi , \\
\mathring{\Box} \, \phi &= m^2 \, \phi , \\
\frac{1}{8\pi} \; \mathring{G}^{\mu\nu} &= \phi^{,\mu} \, \phi^{,\nu} - \frac{1}{2} \, g^{\mu\nu} \left(\phi_{,\alpha} \, \phi^{,\alpha} + m^2 \, \phi^2 \right) ,
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$$\text{where}\quad \mathcal{R}^{\mu\nu\kappa} := \frac{1}{2}\left(\mathcal{P}^{\kappa\mu\nu} + \mathcal{P}^{\kappa\nu\mu} - \mathcal{P}^{\mu\nu\kappa}\right).$$

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\frac{1}{8\pi}\ \overset{\circ}{\mathcal{G}}{}^{\mu\nu} &= 2\left[\frac{\partial \mathcal{L}_{matt}}{\partial g_{\mu\nu}} - \overset{\circ}{\nabla}_\kappa\, \mathcal{R}^{\mu\nu\kappa}\right], \\
\frac{1}{8\pi}\ \overset{\circ}{\mathcal{G}}{}^{\mu\nu} &= 2\frac{\delta \mathcal{L}_{matt}}{\delta g_{\mu\nu}} := \sqrt{|\det g|}\ T^{\mu\nu}\,.
\end{aligned}
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Example: Vector field

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$$\mathcal{L}_{matt} = -\frac{\sqrt{|\det g|}}{2}\left[\left(\overset{\circ}{\nabla}_{\nu} X^{\alpha}\right)\left(\overset{\circ}{\nabla}^{\nu} X_{\alpha}\right) + m^2 X_{\alpha} X^{\alpha}\right].$$

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&\quad + \overset{\circ}{\nabla}_\kappa\left(X^\kappa\, \overset{\circ}{\nabla}{}^{(\mu}\, X^{\nu)} - X^{(\mu}\, \overset{\circ}{\nabla}{}^{\nu)}\, X^\kappa\right).
\end{aligned}$$

Track to affine picture

Is it possible to pass from the metric picture to affine one?

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We have to transform the problematic term with a connection:

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Variation of the metric Lagrangian:

$$\delta \mathcal{L}_g = \left[\frac{\partial \mathcal{L}_{matt}}{\partial g_{\mu\nu}} - \frac{1}{16\pi} \overset{\circ}{\mathcal{G}}{}^{\mu\nu} - \overset{\circ}{\nabla}_\kappa \mathcal{R}^{\mu\nu\kappa} \right] \delta g_{\mu\nu} + $$
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Observation:

Metric appears in two ways: as a control parameter (δg) and momentum (π).

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Remark:

In affine picture metric appears as a momentum!

Track to affine picture

Transformation:

$$\partial_\kappa \left(\mathcal{R}^{\mu\nu\kappa}\,\delta g_{\mu\nu}\right) \quad = \quad \partial_\nu \left(\pi_\kappa{}^{\lambda\mu\nu}\,\delta N^\kappa{}_{\lambda\mu}\right) + \delta\left[\mathring{\nabla}_\kappa\,\mathcal{R}_\sigma{}^{\sigma\kappa}\right],$$

Transformation:

$$\begin{aligned}
\partial_\kappa \left(\mathcal{R}^{\mu\nu\kappa}\,\delta g_{\mu\nu}\right) &= \partial_\nu \left(\pi_\kappa{}^{\lambda\mu\nu}\,\delta N^\kappa{}_{\lambda\mu}\right) + \delta\left[\overset{\circ}{\nabla}_\kappa\,\mathcal{R}_\sigma{}^{\sigma\kappa}\right], \\
N^\kappa{}_{\lambda\mu} &= \frac{16\pi}{\sqrt{|\det g|}}\left[\mathcal{R}_{\lambda\mu}{}^{\kappa} - \frac{1}{2}\,\mathcal{R}_\sigma{}^{\sigma\kappa}\,g_{\lambda\mu} + \right. \\
&\qquad \left. -\frac{2}{3}\left(\delta^\kappa_{(\lambda}\,\mathcal{R}_{\mu)\sigma}{}^{\sigma} - \frac{1}{2}\,\mathcal{R}^\sigma{}_{\sigma(\lambda}\,\delta^\kappa_{\mu)}\right)\right], .
\end{aligned}$$

Track to affine picture

Transformation:

$$\partial_\kappa \left(\mathcal{R}^{\mu\nu\kappa} \, \delta g_{\mu\nu} \right) = \partial_\nu \left(\pi_\kappa{}^{\lambda\mu\nu} \, \delta N^\kappa{}_{\lambda\mu} \right) + \delta \left[\overset{\circ}{\nabla}_\kappa \, \mathcal{R}_\sigma{}^{\sigma\kappa} \right],$$

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$$\left. - \frac{2}{3} \left(\delta^\kappa_{(\lambda} \mathcal{R}_{\mu)\sigma}{}^\sigma - \frac{1}{2} \mathcal{R}^\sigma{}_{\sigma(\lambda} \delta^\kappa_{\mu)} \right) \right], .$$

Result:

$$\delta \mathcal{L}_g = \partial_\kappa \left[p^\kappa \, \delta\phi + \pi_\kappa{}^{\lambda\mu\nu} \, \delta \left(\overset{\circ}{\Gamma}{}^\kappa{}_{\lambda\mu} + N^\kappa{}_{\lambda\mu} \right) \right] + \delta \left[\overset{\circ}{\nabla}_\kappa \, \mathcal{R}_\sigma{}^{\sigma\kappa} \right].$$

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Conlusion:

We have a non-metric connection! $\Gamma = \overset{\circ}{\Gamma} + N$!

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We have an affine description of our theory!

Affine picture

Variation of the affine Lagrangian :

$$\begin{aligned}\delta\mathcal{L}_A &= \partial_\kappa \left[p^\kappa \, \delta\phi + \pi_\kappa{}^{\lambda\mu\nu} \, \delta\Gamma^\kappa{}_{\lambda\mu} \right] = \\ &= \partial_\kappa \left(p^\kappa \, \delta\phi \right) + \pi^{\mu\nu} \, \delta K_{\mu\nu} + \nabla_\nu \pi_\kappa{}^{\lambda\mu\nu} \, \delta\Gamma^\kappa{}_{\lambda\mu} \, .\end{aligned}$$

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Comments:

$K_{\mu\nu}$ is only the symmetric part of the Ricci tensor (for general symmetric connection there also exist a skew-symmetric part!)

Affine picture – example

Vacuum with cosmological constant:

$$\mathcal{L}_g = \frac{\sqrt{|\det g|}}{16\pi}\left(\overset{\circ}{R} - 2\Lambda\right)$$

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Idea:

If we want to take into account whole curvature we have to take the more general momentum of the connection, which symmetric part stays thesame as previously.

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$$K_{\mu\nu} = \Lambda\, g_{\mu\nu} - \frac{1}{\Lambda}\left(F^{\mu\alpha}\, F^{\nu}{}_{\alpha} - \frac{1}{4}\, F_{\alpha\beta}\, F^{\alpha\beta}\, g^{\mu\nu}\right),$$

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Results:

1. Connection is not metric and non-metricity is related to the skew-symmetric part of Ricci curvature $F_{\mu\nu}$ (Weyl concept of unified theory),

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What is next?

I want to apply the last (traceless) part of Riemann curvature part and believe, that it could describe some kind of dark matter.

QUESTIONS?