

---

# An improved discrete Rellich inequality on the half-line

**B. Gerhat<sup>1</sup>, D. Krejčířík<sup>1</sup> and F. Štampach<sup>1</sup>**

*Czech Technical University in Prague<sup>1</sup>  
Department of Mathematics, Faculty of Nuclear Sciences and Engineering*

**E-mail address:** *borbala.gerhat@fjfi.cvut.cz*

## Abstract

The recent results by Keller, Pinchover and Pogorzelski in [2, 3] show that the classical Hardy weight  $1/(4n^2)$  is not optimal in the discrete setting (unlike in the continuous case). An alternative proof of this improved inequality is obtained by a convenient factorisation in [4], which also allows to reprove the optimality of the Keller–Pinchover–Pogorzelski weight.

The main part of this talk concerns the generalisation of the factorisation strategy to the Rellich case. We present the resulting discrete Rellich inequality in [1] which improves upon the classical (in the continuous case optimal) Rellich weight  $9/(16n^4)$ . The optimality of the improved weight is discussed, as well as some limitations of our method.

To conclude, we formulate a conjecture on improved discrete Hardy-type inequalities of higher order and state an open problem arising from our method in the Rellich case.

The talk is based on joint work [1] with D. Krejčířík and F. Štampach.

## References

- [1] B. Gerhat, D. Krejčířík and F. Štampach. An improved discrete Rellich inequality on the half-line. arXiv:2206.11007v1 [math.SP]
- [2] M. Keller, Y. Pinchover and F. Pogorzelski. An improved discrete Hardy inequality. *Amer. Math. Monthly* 125, 4 (2018), 347–350.
- [3] M. Keller, Y. Pinchover and F. Pogorzelski. Optimal Hardy inequalities for Schrödinger operators on graphs. *Comm. Math. Phys.* 358, 2 (2018), 767–790.
- [4] D. Krejčířík and F. Štampach. A Sharp Form of the Discrete Hardy Inequality and the Keller–Pinchover–Pogorzelski Inequality. *Amer. Math. Monthly* 129, 3 (2022), 281–283.