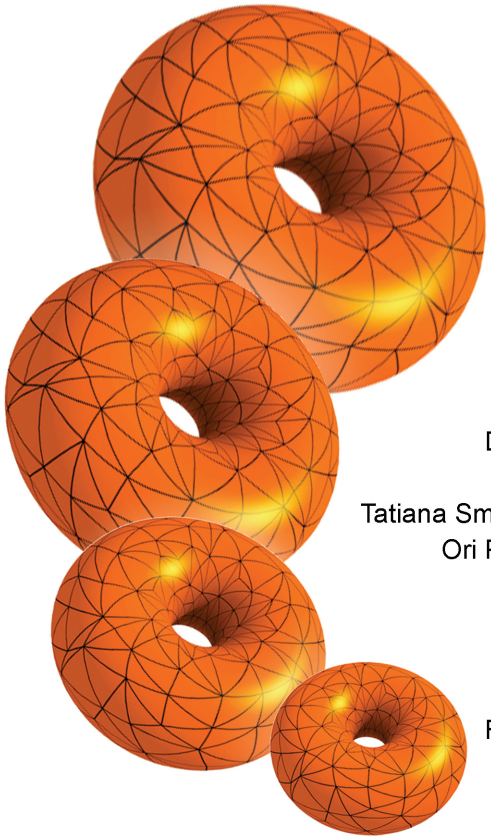
ANALYSIS AND GEOMETRY ON GRAPHS AND MANIFOLDS

International Conference at the University of Potsdam, Germany



July 31st - August 4th 2017

Plenary Speakers:

Colette Anné (Université de Nantes, France)
Jochen Brüning (HU Berlin, Germany)
Józef Dodziuk (Graduate Center, CUNY, USA)
Alexander Grigor'yan (Universität Bielefeld, Germany)
Bobo Hua (Fudan University, China)
Gabor Lippner (Northeastern University, USA)
Jan Maas (IST, Austria)
Dan Mangoubi (Hebrew University of Jerusalem, Israel)
Jun Masamune (Hokkaido University, Japan)
Tatiana Smirnova-Nagnibeda (Université de Genève, Switzerland)
Ori Parzanchevski (Hebrew University of Jerusalem, Israel)
Norbert Peyerimhoff (Durham University, UK)
Yehuda Pinchover (Technion Haifa, Israel)
Peter Stollmann (TU Chemnitz, Germany)
Alexander Teplyaev (University of Connecticut, USA)
Françoise Truc (CNRS Université de Grenoble, France)
Alain Valette (Université de Neuchâtel, Switzerland)
Ivan Veselic (TU Dortmund, Germany)
Wolfgang Woess (TU Graz, Austria)
Andrzej Żuk (Université Paris 7, France)

Organizers:

Matthias Keller (Universität Potsdam)
Daniel Lenz (Friedrich Schiller Universität Jena)
Radosław Wojciechowski (York College and the Graduate Center, CUNY)

Local organizers:

Moritz Gerlach
Michael Schwarz
Melchior Wirth

Sponsors:

SPP 2026 Geometry at infinity
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SIMONS FOUNDATION
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1 General Informations

Conference rooms

All talks will take place in **House 9, Room 1.02**. The coffee breaks are at the foyer of **House 8**. In House 8 you can use **Room 0.64** for mathematical discussions.

Wifi

In the buildings of the University of Potsdam you can use your **eduroam** account. If you do not have an eduroam account, you can get a guest account for the network **UP-Conference** at the registration desk.

Lunch and dinner

A cafeteria is located on campus (**House 12**) where you can get lunch. It is open from 10:00 to 15:00.

Many restaurants where you can get dinner are located near the **Gutenbergstraße**. You can get there by foot from the stations **Dortusstraße** and **Platz der Einheit/West**. These stations can be reached both from the conference venue and from the main station by taking bus 605 or bus 695.

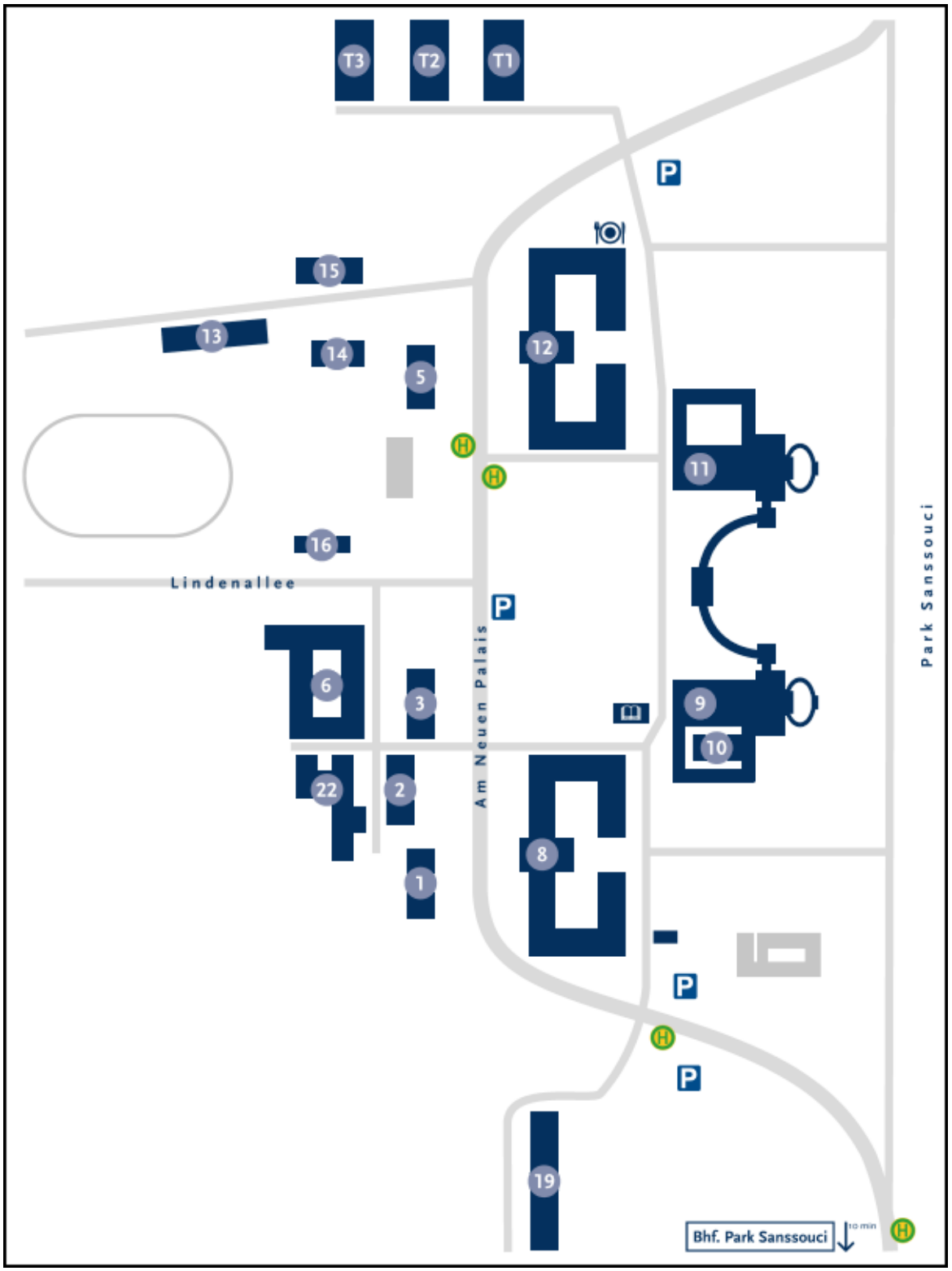
Conference dinner

The conference dinner will take place on **Wednesday** starting **19:00** on the roof of the **Bildungsforum**. You can get there from the bus/tram-stations **Platz der Einheit/West** or **Platz der Einheit/Bildungsforum**.

Public transportation in Potsdam

For transition between the conference venue and the main station you can either take the train (leaving every 30 minutes) from/to the station **Potsdam Park Sanssouci** or you can take the bus (lines 605, 606, and 695). For all public transportation in Potsdam, you can use the tickets for zone **Potsdam A**. These tickets can be bought from ticket machines at the train stations or on the buses.

2 Campus map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3 Program

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
08:50	Welcome				
09:00	Dodziuk	Grigor'yan	Valette	Brüning	Woess
09:40	Parzanchevski	Mangoubi	Teplyaev	Veselic	Zuk
10:20	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
10:50	Nagnibeda	Anné	Maas	Pinchover	Lippner
11:30	Truc	Masamune	Hua	Stollmann	Peyerimhoff
12:10	Lunch	Lunch	Lunch	Lunch	Lunch
13:30	Joyner	Post	Hinz (13:10)	Exner	Pogorzelski (13:10)
14:10	Oh	Güneysu	Alonso-Ruiz (13:40)	Chernyshev	Münch (13:40)
14:40	Torki-Hamza	Joe Chen		Band	
15:10	Coffee break	Coffee break		Coffee break	
15:40	Beckus	Li Chen		Egger	
16:00	McGuirk	Bianchi		Lauret	
16:20	Ayadi	Meinert		Eliaz	
16:40	Coffee break	Coffee break		Coffee break	
17:00	Bombach	Chebbi		Alon	
17:20	Balti	Bersudsky		Medini	

4 Abstracts

On the Nodal Count Distribution of Quantum Graphs

Lior Alon (Technion Haifa, Israel)

I will give a brief introduction to quantum graphs and their nodal count sequence. I will then present the known bounds on the nodal count and define the nodal surplus and the nodal count distribution. I will state two theorems that show an interesting relation between the topology of a graph and its nodal count distribution. This work is based on a joint work with Ram Band and Gregory Berkolaiko.

Power dissipation in fractal AC networks

Patricia Alonso-Ruiz (University of Connecticut, U.S.)

Energy forms on graphs and on fractals can be interpreted in terms of electric linear networks by assuming that current flows between nodes (vertices) connected by resistors (edges). In general, nodes in a passive AC network can be connected by other elements such as inductorsand capacitors. These are called non-dissipativesince no energy is lost when the current flows through them. This type of networks are usually studied in the frequency domainvia Fourier transformation.

In the 60s, Feynman described an infinite passive AC linear network consisting solely of inductors and capacitors that exhibited power dissipation at some frequencies. Based on this idea, we investigate the concept of power dissipation in graphs and fractals associated with passive linear networks with non-dissipative components.

In this talk, we present the so-called Feynman-Sierpinski ladder fractal network, define power dissipation and construct the corresponding induced measure associated with continuous potentials. This measure turns out to be continuous as well as singular with respect to a suitable Hausdorff measure defined on the fractal dust of nodes that constitute the Feynman-Sierpinski ladder.

Norm convergence of the resolvent for wild perturbations

Colette Anné (Université de Nantes, France)

What is the spectral influence on the Laplacian of a complete Riemannian manifold of perturbations such like making a lot of small holes or adding a lot of short and thin handles? Olaf Post had developed, in is book (LNM 2039), a powerful tool concerning the energy quadratic forms, it can be applied in this context and can give convergence results in norm of the resolvent of the operators. [a joint work with O. Post].

On the spectra of discrete Laplacians on forms

Hèla Ayadi (Kairouan University, Tunisia)

In the context of infinite weighted graphs, we consider the discrete Laplacians on 0-forms and 1-forms. Using Weyl's criterion, we prove the relation between the nonzero spectra of these two Laplacians. Moreover, we give an extension of the work of John Lott to characterize their 0-spectrum

Non self-adjoint Laplacian on a directed graph

Marwa Balti (Carthage University, Tunisia & Université de Nantes, France)

We consider the Laplacian on a directed graph with non symmetric edge weights. We analyse some characteristic of this non-selfadjoint Laplacian under a Kirchhoff hypothesis. Moreover we establish an isoperimetric inequality in terms of the numerical range in order to show the lack of the essential spectrum.

Neumann Domains

Ram Band (Technion Haifa, Israel)

The nodal set of a Laplacian eigenfunction forms a partition of the underlying manifold. We present an alternative partition, which is based on the gradient field of the eigenfunction and is called the Neumann domain partition. We point out the similarities and differences between nodal domains and Neumann domains. In particular, we focus on the ground state

property, which holds for nodal domains, and check its validity for Neumann domains. The talk is based on joint works with Sebastian Egger, David Fajman and Alexander Taylor.

This is a first part of a talk which is continued by Sebastian Egger after the coffee break.

Shnol type Theorem for the Agmon ground state

Siegfried Beckus (Technion Haifa, Israel)

The celebrated Shnol theorem [4] asserts that every polynomially bounded generalized eigenfunction for a given energy $E \in \mathbb{R}$ associated with a Schrödinger operator H implies that E is in the L^2-spectrum of H. Later Simon [5] rediscorvered this result independently and proved additionally that the set of energies admiting a polynomially bounded generalized eigenfunction is dense in the spectrum. A remarkable extension of these results hold also in the Dirichlet setting [1, 2].

It was conjectured in [3] that the polynomial bound on the generalized eigenfunction can be replaced by an object intrinsically defined by H, namely, the Agmon ground state. During the talk, we positively answer the conjecture indicating that the Agmon ground state describes the spectrum of the operator H. Specifically, we show that if u is a generalized eigenfunction for the eigenvalue $E \in \mathbb{R}$ that is bounded by the Agmon ground state then E belongs to the L^2-spectrum of H. Furthermore, this assertion extends to the Dirichlet setting whenever a suitable notion of Agmon ground state is available.

References

[1] A. Boutet de Monvel, D. Lenz, and P. Stollmann, *Sch'nol's theorem for strongly local forms*, 189–211, Israel J. Math., vol. 173, 2009.

[2] A. Boutet de Monvel and P. Stollmann, *Eigenfunction expansions for generators of Dirichlet forms*, 131–144, J. Reine Angew. Math. vol. 561, 2003.

[3] B. Devyver, M. Fraas, and Y. Pinchover, *Optimal Hardy weight for second-order elliptic operator: an answer to a problem of Agmon*, 4422–4489, J. Funct. Anal., vol. 266, no. 7, 2014.

[4] È. È. Šhnol', *On the behavior of the eigenfunctions of Schrödinger's equation*, 273–286, Mat. Sb. (N.S.), vol. 42, (84), 1957; erratum, 259, vol. 46, (88), 1957 (Russian).

[5] B. Simon, *Spectrum and continuum eigenfunctions of Schrödinger operators*, 347–355, J. Funct. Anal., vol. 42, no. 3, 1981.

Courant-sharp eigenvalues of Neumann 2-rep-tiles

Michael Bersudsky (Technion Haifa, Israel)

We shall discuss the sharpening of Courant's bound on the number of nodal domains for some specific domains in $\mathbb{R}^n$. In particular we show that our problem is essentialy reduced to a lattice counting problem, which suprisingly admits an elementary and simple solution. This is a joint work with Ram Band and David Fajman.

Laplacian cut-offs

Davide Bianchi (Università degli Studi dell'Insubria, Italy)

Many analytic results in Euclidean setting require the use of compactly supported cut-off functions, essentially to localize differential equations or inequalities or to perform integration by parts arguments. A key feature of d-dimensional Euclidean space is that it is possible to construct cut-offs $\{\phi_R\}$ such that $\phi_R = 1$ on the ball $B_R(o)$, they are supported in the ball $B_{\gamma R}(o)$ and have controlled derivatives up to second order:

$$|\nabla \phi_R| \leq \frac{C}{R}, \qquad |\Delta \phi_R| \leq \frac{C}{R^2},$$

where C is a constant depending only on γ and the dimension. Indeed, such cut-offs can be defined in terms of the distance function r from 0, $r(x) = (\sum_i x_i^2)^{1/2}$, as $\phi_R(x) = \psi(r(x)/R)$ where $\psi : \mathbb{R} \to [0,1]$ is smooth, identically 1 in $(-\infty, 1]$ and vanishes in $[\gamma, +\infty)$, and the properties of ϕ_R listed above depend crucially on the fact that the distance function is proper and satisfies

$$|\nabla r(x)| \leq C, \quad |\Delta r(x)| \leq \frac{C}{r(x)}.$$

The existence of Euclidean cut-offs with the above properties is then a consequence of the fact that distance is a well-behaved proper function on $\mathbb{R}^d$.

In many significant situations it is actually vital to have an explicit uniform decay of $\Delta\phi_R$ in terms of R. We quote, for example, spectral properties of Schrödinger-type operators and the approximation procedures used in the proof of existence, uniqueness and qualitative and quantitative properties of solutions to the Cauchy problem for the porous and fast diffusion equations.

We construct exhaustion and cut-off functions with controlled gradient and Laplacian on manifolds with Ricci curvature bounded from below by a (possibly unbounded) nonpositive function of the distance from a fixed reference point, without any assumptions on the topology or the injectivity radius. Along the way we prove a generalization of the Li-Yau gradient estimate which is of independent interest.

References

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[2] J. Cheeger. Degeneration of Riemannian metrics under Ricci curvature bounds. *Accademia Nazionale dei Lincei. Scuola Normale Superiore.* Lezioni Fermiane (2001).

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[4] R. Schoen, S.T. Yau. Lectures on differential geometry, volume 1. *International press Cambridge* (1994).

Uniqueness for an inverse boundary value problem with Kato potentials

Clemens Bombach (Technische Universität Chemnitz, Germany)

Given an open set Ω with smooth boundary in n-dimensional space and a potential function V on Ω, one can define the Cauchy data space as the set of Dirichlet and Neumann boundary data corresponding to solutions of the stationary Schrödinger equation $-\Delta u + Vu = 0$. If V is in $L^{n/2}$, it is known that V is uniquely determined by its associated Cauchy data space. We present a related result in dimension three where it is assumed that V is in the Kato class. This potential class contains all L^p spaces for $p > 3/2$ but is not contained in $L^{3/2}$.

Analysis on smooth Thom-Mather spaces

Jochen Brüning (Humboldt-Universität zu Berlin, Germany)

The discrete Laplacian of a 2-simplicial complex

Yassin Chebbi (Université de Nantes, France)

We introduce the notion of oriented faces especially triangles in an oriented connected locally finite graph. This framework permits to define the Laplacian on the structure of a 2-simplicial complex then we discuss its essential self-adjointness through a geometric hypothesis.

Limit shape universality in cellular automata models on the Sierpinski gasket

Joe Chen (Colgate University, U.S.)

It has been conjectured that on any state space, the growing clusters associated with the four cellular automata models—internal diffusion-limited aggregation (IDLA), rotor-router aggregation, divisible sandpiles, and abelian sandpiles—have the same limit shape. This conjecture is far from being proven. Even on $\mathbb{Z}^d$, while it has been proved that the limit shape is an Euclidean ball in the case of IDLA, rotor-router aggregation, and divisible sandpiles, the case of abelian sandpiles remains open.

I will explain that on the infinite graphical Sierpinski gasket (SG), when particles are launched from the corner vertex o of SG, the cluster in each of the four models fills balls (centered at o in the graph metric) with (asymptotically) precise rates. Thus there is a kind of "limit shape universality" on SG. We do not know the extent to which such phenomenon holds on other graphs, but some of our proof methods may hint at possible mechanisms underlying the universality.

This talk covers work initiated with Wilfried Huss (TU Graz), Ecaterina Sava-Huss (TU Graz), and Sasha Teplyaev (UConn), and culminates with a joint work with Jonah Kudler-Flam (UChicago).

Boundedness of Riesz transform without Gaussian heat kernel estimates

Li Chen (Instituto de Ciencias Matemáticas, Madrid, Spain)

We obtain that the Riesz transform is L^p bounded for $1 < p < 2$ on Riemannian manifolds and graphs under the volume doubling property and a sub-Gaussian heat kernel upper bound. In the particular case of Vicsek manifolds and graphs, we show that the reverse Riesz transform doesn't hold for $1 < p < 2$ and hence the Riesz transform is not L^p bounded for $p > 2$ by duality, which is strikingly different from the Euclidean case.

A dynamical system on metric graphs and hybrid spaces and number-theoretical problems

Vsevolod Chernyshev (National Research University "Higher School of Economics", Moscow, Russia)

Let us consider a finite compact metric tree and the following dynamical system on it. Let one point move along the graph at the initial moment of time. If K points come to the interior vertex of valence V at the same time then V points are released, i.e. one point corresponds to one edge. Reflection occurs in vertices of valence one. Time for passing each individual edge is fixed. The problem is to analyze the asymptotic behavior of the number $N(T)$ of such points on the graph as time T increases. Such dynamical system emerges while considering the Cauchy problem for the time-dependent Schrodinger equation on metric graphs and hybrid spaces.

In the case of linearly independent lengths the problem is related to the problem of counting the lattice points. An asymptotic expansion for $N(T)$ using Barnes' multiple Bernoulli polynomials (also known as Todd polynomials) was found. The second term is given by a quadratic form of edge travel times. The tree structure is uniquely determined by this form.

In the case of rational lengths $N(T)$ reaches a plateau at a certain moment of time (stabilization time). The problem of finding the stabilization time is related to the problem of finding a Frobenius number and the Skolem-Malher-Lech theorem.

Let us consider a similar problem for the hybrid space constructed by gluing a segment to a Riemannian manifold with positive topological entropy h. In this case the problem of finding the number of moving points on the segment is related to an abstract prime number theorem. $N(T)$ grows exponentially and the growth rate depends on h.

Tunnels of positive scalar curvature

Jozef Dodziuk (Graduate Center CUNY, U.S.)

Gromov and Lawson (1980) and independently, in dimension less than or equal to 7, Schoen and Yau (1979) proved that connected sums of manifolds with positive scalar curvature admit metrics of positive scalar curvature. They constructed tunnels diffeomorphic to $S^{n-1} \times [0, 1]$ that can be glued smoothly into n-spheres of constant sectional curvature. We revisit the construction and show tunnels of arbitrarily small volume and diameter can be created. These tunnels are then used to construct examples of sequences of compact manifolds with positive scalar curvature converging to spaces that fail to have positive scalar curvature in a certain generalized sense.

Neumann Domains (II)

Sebastian Egger (Technion Haifa, Israel)

This talk is based on Ram Band's talk on Neumann domains and I concentrate on some techniques of the proof for the ground state property for star like Neumann domains for Laplacian eigenfunctions on the torus. In particular, I present a novel rearrangement trick which involves not a disc but a sector allowing as some freedom in the sector angle, but limited by spectral constraints inherited from our considered eigenfunction. This trick allows us to compare eigenvalues for different eigenfunctions and eventually to prove the ground state property for 'large eigenvalues'.

On the Essential Spectrum of Schrödinger Operators on Trees

Latif Eliaz (Hebrew University of Jerusalem, Israel)

It is known that the essential spectrum of a Schrödinger operator H on $\ell^2(\mathbb{N})$ is equal to the union of the spectra of right limits of H. The natural generalization of this relation to $\mathbb{Z}^n$ is known to hold as well. In this talk we study the possibility of generalizing this characterization of $\sigma_{ess}(H)$ to trees. We give indications for the failure of the general statement in this case, while presenting a natural family of models where it still holds. This is joint work with Jonathan Breuer.

Periodic quantum graphs may exhibit uncommon spectra

Pavel Exner (Czech Academy of Sciences, Prague, Czech Republic)

A common wisdom says that the spectrum of a periodic quantum graph consists of absolutely continuous bands, typically infinitely many of them, and possibly infinitely degenerate 'Dirichlet' eigenvalues. The main aim of the talk is to show that there are other options. Using two simple examples, a chain of loops and a rectangular lattice, I am going to show that (i) a chain in a homogeneous magnetic field can have no absolutely continuous spectrum at all, (ii) a chain in a linear magnetic field can have a spectrum of a fractal nature, and (iii) even without any external field a lattice can have a finite number of spectral gaps in analogy with Bethe-Sommerfeld behaviour of the 'usual' Schrödinger operators. The last two effects depend on the number- theoretic properties of the model parameters.

On semi-linear elliptic inequalities on Riemannian manifolds

Alexander Grigor'yan (Universität Bielefeld, Germany)

We consider a problem of existence of non-trivial positive solutions to certain semi-linear elliptic inequalities on complete Riemannian manifolds. The results about existence and non-existence are stated in terms of the volume growth function and Green function estimates.

Kac regular sets and Sobolev spaces in geometry and probability

Batu Güneysu (Humboldt-Universität zu Berlin, Germany)

Assume U is an open subset of a Riemannian manifold M and w is a potential on M such that the Schrödinger operator $-\Delta + w$ is bounded from below in the Hilbert space $L^2(M)$. With $W_0^{1,2}(M; w)$ the natural form domain of $-\Delta + w$, the aim of this talk is answer the following question:

Under which assumption on U are those functions f from $W_0^{1,2}(M; w)$ that vanish almost everywhere in the complement of U automatically in $W_0^{1,2}(U; w)$?

It turns out that this property provides a highly subtle local boundary regularity assumption on U, which (following Stroock, Herbst and Zhao in the Euclidean case), we call "Kac regularity". This property can be characterized probabilistically. In fact, we treat more general covariant Schrödinger operators, allowing to prove new harmonicity results for Dirac spinors on subsets having a possibly very singular boundary. This is joint work with Francesco Bei and Stefano Pigola.

Differential forms on products of fractals

Michael Hinz (Universität Bielefeld, Germany)

During the recent years differential one-forms on fractal spaces have been studied by various authors. In this talk we consider products of fractals, explain a way to introduce corresponding higher order differential forms and discuss how they can be approximated by antisymmetric functions on products of graphs. Our guiding example will be two-forms on the product of two Sierpinski gaskets. The talk is based on work in progress with Dan Kelleher (Alberta).

Combinatorial curvature for planar graphs

Bobo Hua (Fudan University, China)

The combinatorial curvature of a planar graph is defined as the generalized Gaussian curvature of its polygonal surface with a piecewise flat metric. We will show that the total curvature of a planar graph, whose faces are isometric to regular polygons in the Euclidean plane, with nonnegative combinatorial curvature is an integral multiple of $\frac{1}{6}\pi$. This is a joint work with Yanhui Su.

Quotients of finite-dimensional operators by symmetry representations

Christopher Joyner (Queen Mary University of London, U.K.)

When a finite-dimensional operator is invariant under some symmetry group G then it admits, what is termed, a quotient operator with respect to a certain representation of G. For example, if one has a reflection symmetry then a certain quotient operator allows to select those eigenfunctions which are either even or odd under reflection. In general, the quotient operator contains the information contained in the original operator corresponding to a suitable representation of G. Such operators were first introduced in the context of isospectral graphs and manifolds and defined in terms of their spectral properties. Here we provide an alternative non-spectral definition that allows us to generalise previous quotient constructions, account for fixed points in the system, consider non-Hermitian operators and provide tools for computational purposes. This is joint work with R. Band, G. Berkolaiko and W. Liu.

One-norm spectrum of a lattice

Emilio Lauret (Humboldt Universität zu Berlin, Germany)

In 1964, John Milnor gave the first example of isospectral non-isometric compact Riemannian manifolds. To do this, he related the spectrum of the Laplace operator on a torus, and the (Euclidean) norm of the vectors of the (corresponding) dual lattice. Consequently, a pair of lattices with the same theta function induces a pair of isospectral tori.

In this talk, we will introduce a new relation between the spectrum of a lens space (a sphere over a cyclic group), and the one-norm (sum of the absolute values of the entries) of the vectors in an associated lattice. We associate to each lattice, the one-norm generating

function defined as follows: the power series whose k-th term is the number of vectors in the lattice with one-norm equal to k.

We will show that two lens spaces are isospectral if and only if their corresponding lattices have the same one-norm generating function. Furthermore, we will prove that the generating function is a rational function, and consequently, a finite part of the spectrum determines the whole spectrum.

This is a joint work with Roberto Miatello and Juan Pablo Rossetti.

Quantum state transfer on graphs

Gabor Lippner (Northeastern University, U.S.)

Transmitting quantum information losslessly through a network of particles is an important problem in quantum computing. Mathematically this amounts to studying solutions of the discrete Schrödinger equation $\frac{d}{dt}\phi = \mathrm{i}\, H\phi$, where H is typically the adjacency or Laplace matrix of the graph. This in turn leads to questions about subtle number-theoretic behaviour of the eigenvalues of H.

It has proven to be difficult to find graphs which support such information transfer. I will talk about recent progress in understanding what happens when one is allowed to apply magnetic fields (that is, adding a diagonal matrix to H) to the system of particles.

(Joint work with Mark Kempton, S-T Yau, Krystal Guo, and Chris Godsil.)

Gradient flows and entropy inequalities for dissipative quantum systems

Jan Maas (IST Wien, Austria)

We present a new class of transport metrics for density matrices, which can be viewed as non-commutative analogues of the 2-Wasserstein metric. With respect to these metrics, we show that dissipative quantum systems can be formulated as gradient flows for the von Neumann entropy under a detailed balance assumption. We also present geodesic convexity results for the von Neumann entropy in several interesting situations. These results rely on an intertwining approach for the semigroup combined with suitable matrix trace inequalities. This is joint work with Eric Carlen.

Harmonic functions - positivity and convexity

Dan Mangoubi (Hebrew University of Jerusalem, Israel)

We present a sequence of positive quadratic forms associated with a harmonic function on an Abelian group. We show how the positivity property recovers the Liouville property and we prove a Three Spheres theorem in terms of random walks. The talk is based on a joint work with Gabor Lippner.

H-convergence on Riemannian manifolds

Jun Masamune (Hokkaido University, Japan)

In this talk we will show that the classical H-convergence on Euclidean space generalizes to some Laplace type operators on Riemannian manifolds. Joint with H. Hoppe and S. Neukamm.

Γ-Calculus of Cones

Zachary McGuirk (Graduate Center CUNY, U.S.)

In this talk we find a Poincaré inequality for functions acting on the vertices of a finite graph by analyzing the $CD(K, N)$-inequality for a cone over the vertices and restricting our attention to the cone point.

The magnetic bottle on graphs

Zied Medini (University of Carthage, Tunisia & University of Orléans, France)

We recall the definition of the discrete magnetic Laplacian and its domain. We try to discuss the phenomenon of the magnetic bottle on a type of graph that we introduce. Indeed, the magnetic field has the effect of emptying the essential spectrum.

The viscous Burgers Equation on locally metric spaces

Melissa Meinert (Universität Bielefeld, Germany)

In this talk, we are going to formulate an abstract version of the viscous Burgers Equation on locally metric spaces. In particular, we discuss existence of weak solutions by using the Cole-Hopf Transformation which transforms a nonlinear equation into a linear parabolic equation. Starting with a regular symmetric Dirichlet form, we develop a suitable notation of abstract derivations which we need to impress the convection term. As an application, we also consider the viscous Burgers equation on metric graphs.

Relations between Ricci curvature, coverings, path homology, homotopy groups and cycle bases on graphs

Florentin Münch (Universität Potsdam, Germany)

On a finite graph with positive Ricci curvature, the Bonnet-Myers diameter bound implies that there exists no infinite connected graph covering preserving 3- and 4-cycles. We show how the existence of such a graph covering relates to the path homology by Grigor'yan, Lin, Muranov and Yau and to a certain homotopy group of the underlying graph. This homotopy group has been used to investigate gain graphs and biased graphs. Using this concept, we introduce gain and biased cycle bases and show, how these cycle basis notions classify into the cycle basis hierarchy developed by Liebchen and Rizzi.

Infinite Ramanujan graphs

Tatiana Nagnibeda (Université de Genève, Switzerland)

A graph is Ramanujan if its spectral gap is maximal possible and hence coincides with that of the graph's covering tree. This definition applies equally well to finite and infinite graphs, but Ramanujan property has being mostly studied for finite graphs, as in this case Ramanujan graphs are the best possible expanders. In this talk, based on a joint work with Vadim Kaimanovich, I will discuss some properties and open conjectures of infinite Ramanujan graphs.

Combinatorial curvature and isoperimetric constants on infinite planar graphs

Byung-Geun Oh (Hanyang University, Korea)

Combinatorial curvature on planar graphs is a discrete analogue of Gaussian curvature on Riemannian 2-manifolds, and it has become an important concept in the study of

some structural problems in graph theory. In this talk we will mostly concern negativity conditions of combinatorial curvatures and its relations to isoperimetric constants on planar graphs, and especially discuss lower bounds of isoperimetric constants on planar graphs whose vertex and face degrees are at least p and q, where p, q are natural numbers such that $1/p + 1/q < 1/2$.

Random walks on Ramanujan digraphs and complexes

Ori Parzanchevski (Hebrew University of Jerusalem, Israel)

Ramanujan graphs are finite graphs which behave spectrally like an inifinite tree. In the same spirit, Ramanujan digraphs are directed graphs which behave like an infinite directed tree. I will explain what these are, how can they be constructed, and how they relate to the study of Random walks on Ramanujan complexes, which are a high dimensional analogue of Ramanujan graphs. Based on joint works with Eyal Lubetzky, Alex Lubotzky and Peter Sarnak.

Some recent results for Ollivier-Ricci curvature on graphs

Norbert Peyerimhoff (Durham University, U.K.)

Ollivier proposed in 2009 a curvature notion of Markov chains on metric spaces, based on optimal transport of probability measures associated to a random walk. In the special setting of graphs, this concept provides a curvature on the edges and depends on an idleness parameter of the random walk. Lin, Lu, and Yau modified this notion in 2011. In this talk, I will recall this curvature notion and present some specific results, which are based on joint work with D. Bourne, D. Cushing, R. Kangaslampi, Sh. Liu, and F. Muench.

Optimal Hardy-type inequality for second-order elliptic operator: the continuum case

Yehuda Pinchover (Technion Haifa, Israel)

For a general subcritical linear second-order elliptic operator P with real coefficients defined on a domain M in $\mathbb{R}^n$ (or on a noncompact manifold M), We construct a Hardy-type weight W which is optimal in the following natural sense:

- For all $\lambda \leq 1$, $(P - \lambda W) \geq 0$ in M (in the sense that the equation $(P - \lambda W)u = 0$ admits positive solution in M),
- For $\lambda = 1$, the operator $(P - \lambda W)$ is null-critical in M,
- For any $\lambda > 1$, and any neighborhood of infinity $G \subset M$, the operator $(P - \lambda W)$ is not nonnegative on G.
- If P is symmetric and $W > 0$, then the spectrum and the essential spectrum of the operator $W^{-1}P$ are equal to $[1, \infty)$.

Our method is based on the theory of positive solutions and applies to both symmetric and nonsymmetric operators on noncompact manifolds. The constructed weight W is given by an explicit simple formula involving two positive solutions of the equation $Pu = 0$.

This is a joint work with Baptiste Devyver and Martin Fraas.

Optimal Hardy inequalities on graphs

Felix Pogorzelski (Technion Haifa, Israel)

The talk is devoted to sharp Hardy inequalities for Laplace (type) operators on weighted graphs. We give sufficient criteria for the existence of Hardy weights in terms of positive superharmonic functions. The method is based on a discrete version of the supersolution construction which has been studied before in the context of elliptic Schrödinger operators on continuous spaces. We also intend to shed some light on optimality criteria and - time permitting - say a word or two about the proofs. Joint work with Matthias Keller and Yehuda Pinchover.

Spectral gaps and discrete magnetic Laplacians

Olaf Post (Universität Trier, Germany)

In this talk we present a simple geometric condition that guarantees the existence of spectral gaps of the discrete Laplacian on periodic discrete graphs. The basic tool is a spectral bracketing on the level of the finite quotient graph using the concept of "virtualising" edges and vertices. We verify in several examples how the method works.

Bounds for Schrödinger semigroups on manifolds: the role of curvature and applications

Peter Stollmann (Techniche Universität Chemnitz, Germany)

We present recent joint work with C. Rose on geometrically motivated Schrödinger operators. The applications concern the space of harmonic forms. The main issue is to achieve the respective results under weak assumptions on Ricci curvature.

Spectral analysis on non-smooth spaces

Alexander Teplyaev (University of Connecticut, U.S.)

The talk will outline recent achievements and challenges in spectral and stochastic analysis on non-smooth spaces that are very singular, but can be approximated by graphs or manifolds. In particular, the talk will present two of most interesting examples that are

currently under investigation. One example deals with the spectral analysis of the Laplacian on the famous basilica Julia set, the Julia set of the polynomial $z^2 - 1$. This is a joint work with Luke Rogers and several students at UConn. The other example deals with spectral analysis for the canonical diffusion on the pattern spaces of an aperiodic Delone set. This is a joint work with Patricia Alonso-Ruiz, Michael Hinz and Rodrigo Trevino.

On Discrete Hodge-Laplacians

Nabila Torki-Hamza (University of Kairouan & Science Faculty of Bizerte, Tunisia)

Topological Resonances on Quantum Graphs

Françoise Truc (CNRS Université de Grenoble, France)

We consider metric graphs which consist of a finite graph with some leads attached to some vertices. To this metric graph is associated a Laplacian using the Kirchhoff conditions. We are interested in resonances close to the real axis, which are the most important in physics. We show that there is a dichotomy between graphs which can have eigenfunctions with compact support for some particular metrics and the other ones, which are trees with at most one vertex of degree one. In the first case, there are many resonances close to the real axis and we are able to say something on their asymptotics, while in the second one, there is a gap which is an invariant associated to the graph.

Coarse disjoint unions of towers of coverings

Alain Valette (Université de Neuchâtel, France)

We consider coarse disjoint unions of towers of coverings of a given Riemannian manifold, up to coarse equivalence. We prove the following rigidity result: let X, Y be irreducible Riemannian symmetric spaces of rank ≥ 2, let Γ, Λ be co-compact lattices on X, Y respectively. Fix in Γ (resp. Λ) a sequence (Γ_k) (resp. (Λ_k)) of finite-index normal subgroups decreasing to the identity, and consider the coarse disjoint unions $\coprod_k X/\Gamma_k$ and $\coprod_k Y/\Lambda_k$. If they are coarsely equivalent, then X and Y are isometric. The proof goes by replacing X/Γ_k by the Cayley graph of Γ/Γ_k. This is joint work with Ana Khukhro.

Glivenko–Cantelli Theory, Banach-space valued Ergodic Theorems and uniform approximation of the integrated density of states

Ivan Veselic (Technische Universität Dortmund, Germany)

The integrated density of states is the cumulative distribution function of the spectral measure of a random ergodic Hamiltonian. It can be approximated by cumulative distribution functions associated to finite volume Hamiltonians. We study uniform convergence for this approximation in the case where the Hamiltonian is defined on an Euclidean lattice, or more generally, on a discrete amenable group. We obtain a convergence estimate which can be seen as a special case of a Banach space valued Ergodic Theorem. Our proof relies on multivariate Glivenko–Cantelli Theorems. (This is joint work with Christoph Schumacher and Fabian Schwarzenberger.)

Heat kernel oscillations for isotropic random processes on ultrametric spaces

Wolfgang Woess (Technische Universität Graz, Austria)

The family of hierarchical Laplacians on an ultrametric space is introduced and the associated isotropic semigroups of Markov transition operators are displayed. Focussing on the space-homogeneous situation, typical classes of such processes and operators are studied in more detail, where the return probabilities exhibit periodic oscillations.

This comprises the operator fractional derivative (Taibleson Laplacian) on the p-adic numbers and random walks on locally finite groups such as the infinite sum of copies of a finite group and the infinite symmetric group.

This is joint work with Alexander Bendikov (Wroclaw) and Wojciech Cygan (Wroclaw and Graz), based on previous joint work with Bendikov and Alexander Grigor'yan (Bielefeld).

Spectra of discrete KdV

Andrzej Zuk (Université Paris 7, France)

The discrete analogues of the KdV equation are the box-ball systems. Their evolution can be described by automata to which we associate random walk operators. Spectral properties of these operators are related to L^2 Betti numbers of closed manifolds.