On Discrete Hodge-Laplacians

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AGGM Potsdam, July 31st 2017

Joint work with C. Anné & GS-Team: Hèla, Marwa, Yassin, Zied

GS-Team

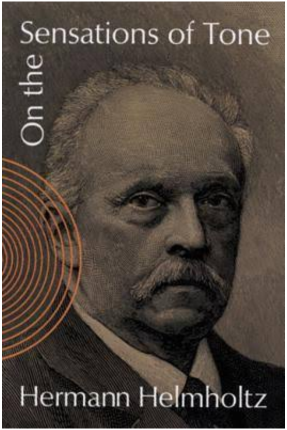
Graph & Spectra Team at Berlin on July 29th 2017

Introduction & Motivation

We would like to define on weighted graphs and on weighted triangulations, the discrete **Gauß-Bonnet** operators and the discrete **Helmhlotzians** or vector Laplace operators .

We present here some of the GS-team main results dealing with essential self-adjointeness and results on spectra.

Hermann von Helmholtz



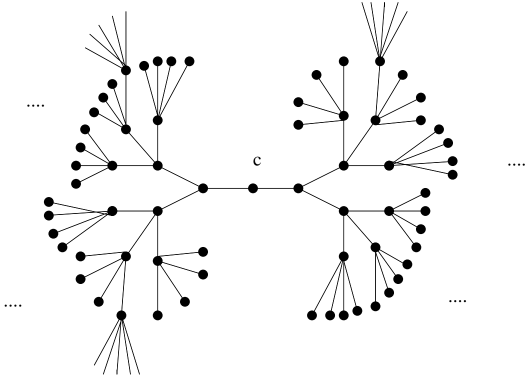
8/ 31/ 1821 , **Potsdam**—— 9/ 8/ 1894 , **Berlin-Charlottenburg**

We can see a weighted graph and a weighted triangulation as a generalization of an electrical network and resp. a tessalation.

They appear in the finite case in many domains and applications like :

- computer vision
- Hodge ranking
- decomposition of finite games in game theory....

An infinite graph



An infinite tree with regular increasing valence (from the vertex c).

References of our project of essential self-adjointeness and spectra

C. Anné & N. T-H
The Gauss-Bonnet operator of an infinite graph ; Analysis and Mathematical Physics 5 (2), 137-159 (2015)

H. Ayadi
Semi-Fredholmness of the Gauss-Bonnet operator ; Filomat, 31 :7, 1909–1926, (2017)

H. Ayadi
Spectra of Laplacians on forms on an infinite graph ; Operator and Matrices, 11 :2 , 567–586 (2017)

Y. Chebbi
The discrete Laplacian of a 2-simplicial complex, accepted and reviewed in Potential Analysis, 26 p, (2017)

References of our project of non self-adjoint Laplacian ...

M. Balti
Non self-adjoint Laplacians on a directed graph ; accepted and reviewed in Filomat, 26 p, (2017)

M. Balti
On the eigenvalues of weighted directed graphs ; Complex Analysis and Operator Theory, 11 :6, pp 1387–1406, (2017)

Other Work In Progress :

Z. Medini
A magnetic botle on a linear like graph ; preprint (2017).

Plan

Set-up & Preliminaries

Set-up & Preliminaries

A graph resp triangulation K is considered as a k-dimensional simplicial complex, $k=1$ resp $k=2$.

- $\mathcal{V}$ denotes the set of its *vertices*, $\mathcal{E}$ the set of its *oriented edges* seen as a subset of $\mathcal{V}\times\mathcal{V}$ and $\mathcal{F}$ the set of its *oriented faces*.

$$K \equiv (\mathcal{V},\mathcal{E})\text{or}(\mathcal{V},\mathcal{E},\mathcal{F}).$$

- We assume that $\mathcal{E}$ is symmetric without loops :

$$v\in\mathcal{V}\Rightarrow(v,v)\notin\mathcal{E},\quad (v_1,v_2)\in\mathcal{E}\Rightarrow(v_2,v_1)\in\mathcal{E}.$$

- The graph is indirected and oriented. Choosing an orientation of $\mathcal{E}$ consists of defining a partition of $\mathcal{E}$:

$$\mathcal{E}^+\sqcup\mathcal{E}^-=\mathcal{E}$$
$$(v_1,v_2)\in\mathcal{E}^+\iff(v_2,v_1)\in\mathcal{E}^-.$$

For $e=(v_1,v_2)\in\mathcal{E}, e^+=v_2,\ e^-=v_1,\ -e=(v_2,v_1)$.

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For $e = (v_1, v_2) \in \mathcal{E}$, $e^+ = v_2$, $e^- = v_1$, $-e = (v_2, v_1)$.

- d_x : the degree of a vertex $x \in \mathcal{V}$ is the cardinal of the set $\{e \in \mathcal{E};\ e^- = x\}$.
- K is with bounded degree, if there exists N, for any x in V, $d_x \leq N$.
- If $\mathcal{V}$ is finite, K is called a finite graph.
- Otherwise K is an infinite graph .

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Paths

A *path* between two vertices x, y in $\mathcal{V}$ is a finite set of edges $e_1, \ldots, e_n, n \geq 1$ such that

$$e_1^- = x,\ e_n^+ = y \ \text{ and, if } \ n \geq 2,\ \forall j,\ \ 1 \leq j \leq (n-1) \Rightarrow e_j^+ = e_{j+1}^-.$$

Γ_{xy} denotes the set of the paths from the vertex x to the vertex y.

Notice that

- each path has a begining and an end.
- an edge is a path.

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Conventions on the graph or the triangulation K

- All the faces are triangles
- K is connected : two vertices are always related by a path.
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Plan

Metrics on a graph

- A *metric* is an even function

$$a : \mathcal{E} \to \mathbb{R}_+^*,$$

it defines a distance on the graph K in the following way. One first defines the *length of a path* $\gamma = (e_1, \ldots, e_n)$

$$l_a(\gamma) = \sum_{j=1}^{n} \sqrt{a(e_j)}.$$

Then the *metric distance* between two vertices x, y is given by

$$d_a(x, y) = \inf_{\gamma \in \Gamma_{xy}} l_a(\gamma).$$

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Plan

Discrete functions and forms

- The $0-$ forms or $0-$cochains are just scalar functions on $\mathcal{V}$ or vertex functions.
- The $1-$ forms or $1-$cochains are skew-symmetric edge flows on $\mathcal{E}$.
- The $2-$ forms or $2-$cochains are triangular-curl flows on $\mathcal{F}$.

- $C^0(K) = \mathbb{C}^{\mathcal{V}}$,
- $C^1(K) = \{\varphi : \mathcal{E} \to \mathbb{C}, \varphi(-e) = -\varphi(e)\}$.
- $C^2(K) = \{\phi : \mathcal{F} \to \mathbb{C}, \phi(-\varpi) = -\phi(\varpi)\}$.
- The sets of cochains with finite support are denoted respectively by $C_c^0(K)$, $C_c^1(K), C_c^2(K)$.

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Hilbert structures

The weights :

- $c : \mathcal{V} \to \mathbb{R}_+^*$
- $r : \mathcal{E} \to \mathbb{R}_+^*, \quad r$ even, $\quad r(-e) = r(e)$
- $s : \mathcal{F} \to \mathbb{R}_+^*, \quad s(-\varpi) = s(\varpi)$

define scalar products :

$$< f, g >= \sum_{v \in \mathcal{V}} c(v) f(v) \bar{g}(v) \quad \text{for } f, g \in C_c^0(K)$$

$$< \phi, \psi >= \frac{1}{2} \sum_{e \in \mathcal{E}} r(e) \phi(e) \bar{\psi}(e) \quad \text{for } \phi, \psi \in C_c^1(K)$$

Finally the Hilbert spaces

$$L_2(\mathcal{V}) := \overline{C_c^0(K)}, \quad L_2(\mathcal{E}) := \overline{C_c^1(K)}, \quad L_2(\mathcal{F}) := \overline{C_c^2(K)}.$$

Plan

Operators on weighted graphs or triangulations

The operator d_0

Definition

- *The difference operator is the linear operator*

$$\mathrm{d}_0 : C_0^0(K) \to C_0^1(K),$$

given by

$$\mathrm{d}_0(f)(e) = f(e^+) - f(e^-).$$

- *The coboundary operator* δ_0 *is the formal adjoint of* d_0. *Thus it satisfies*

$$< \mathrm{d}_0 f, \phi > = < f, \delta_0 \phi >$$

for all $f \in C_0^0(K)$ *and* $\phi \in C_0^1(K)$.

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The operator δ_0

Lemma

The coboundary operator $\delta_0 : C_c^1(K) \to C_c^0(K)$, is defined by the formula

$$\delta_0(\phi)(x) = \frac{1}{c(x)} \sum_{e, e^+ = x} r(e)\phi(e).$$

Remark

The operator d_0 is defined in all $C^0(K)$, but to define δ_0 in all $C^1(K)$, we need to assume the graph K to be locally finite.
This assumption could be weakened by taking the edge weights r summable around each vertex, as considered by Keller-Lenz.

M. Keller & D. Lenz
Dirichlet forms and stochastic completeness of graphs and subgraphs, ; J. reine angew. Math., ***666***, 189–223 (2012)

The operators d_1 and δ_1

The exterior derivative : $d_1 : C_c^1(K) \longrightarrow C_c^2(K)$ is given by

$$d_1(\psi)(x, y, z) = \psi(x, y) + \psi(y, z) + \psi(z, x).$$

The co-exterior derivative : $\delta_1 : C_c^2(K) \longrightarrow C_c^1(K)$ is the formal adjoint of d_1 .

Y. Chebbi
The discrete Laplacian of a 2-simplicial complex, accepted and reviewed in Potential Analysis, 26 p, (2017)

The Gauss-Bonnet operator

The graph Gauss-Bonnet operator is the endomorphism

$$D = \mathrm{d}_0 + \delta_0 : C_c^0(K) \oplus C_c^1(K) \circlearrowleft .$$

It is a symmetric operator and of Dirac type.

For $(f, \psi) \in C_c^0(K) \oplus C_c^1(K)$,

$$D(f, \psi) = \mathrm{d}_0 f + \delta_0 \psi$$

It is a generalization of the Dirac operator studied on the graph $\mathbb{Z}$ by Golenia and Haugomat.

S. Golénia & T. Haugomat,
On the A.C. spectrum of 1D discrete Dirac operator, ; ArXiv 1207.3516, (2012), to appear in Meth. Funct. An. Top.

The Gauss-Bonnet operator

The triangulation Gauss-Bonnet operator is the endomorphism

$$T : C_c^0(K) \oplus C_c^1(K) \oplus C_c^2(K) \circlearrowleft$$

defined by :

$$T(f,\varphi,\phi) = (\delta_0\varphi, d_0 f + \delta_1\phi, d_1\varphi).$$

For $(f,\varphi,\phi) \in C_c^0(K) \oplus C_c^1(K) \oplus C_c^2(K)$,
It is a symmetric operator and it is of Dirac type.

The Laplacian and the Helmholtzian

The graph Laplacian is

$$\Delta = D^2 : C_0^0(K) \oplus C_0^1(K) \circlearrowleft .$$

This operator preserves the direct sum $C_0^0(K) \oplus C_0^1(K)$, so we can write

$$\Delta = \Delta_0 \oplus \Delta_1.$$

The triangulation Laplacian or Helmholtzian is the operator

$$\mathcal{L} := T^2 : C_c^0(K) \oplus C_c^1(K) \oplus C_c^2(K) \circlearrowleft$$

given by

$$\mathcal{L}(f, \varphi, \phi) = (\delta_0 d_0 f, (d_0\delta_0 + \delta_1 d_1)\varphi, d_1\delta_1\phi).$$

We can write

$$\mathcal{L} := \mathcal{L}_0 \oplus \mathcal{L}_1 \oplus \mathcal{L}_2$$

Main results

Plan

$\chi-$completeness

A sufficient geometric condition for Essential self-adjointness is $\chi-$*Completeness* of the graph : it means that there exists a growing sequence of finite sets $(B_n)_{n\in\mathbb{N}}$ such that $\mathcal{V} = \overset{\uparrow}{\cup} B_n$ and there exist related functions χ_n satisfying the following three conditions

(i) $\chi_n \in C_0^0(K)$, $0 \leq \chi_n \leq 1$

(ii) $v \in B_n \;\Rightarrow\; \chi_n(v) = 1$

(iii) $\exists C > 0, \forall n \in \mathbb{N},\, x \in \mathcal{V},\, \frac{1}{c(x)} \sum_{e,e^{\pm}=x} r(e)\mathrm{d}\chi_n(e)^2 \leq C.$

C. Anné & N. T-H
The Gauss-Bonnet operator of an infinite graph ; Analysis and Mathematical Physics 5 (2), 137-159 (2015) ; preprint in 2013.

$\chi-$complete triangulation

Definition

A triangulation is $\chi-$complete, if the two following conditions are satisfied :

- *the corresponding graph is $\chi-$complete.*
- $\exists M > 0,\ \forall n \in \mathbb{N},\ e \in \mathcal{E}$, *such that*

$$\frac{1}{r(e)} \sum_{x \in \mathcal{F}_e} s(e,x) |d^0 \chi_n(e^-, x) + d^0 \chi_n(e^+, x)|^2 \leq M.$$

Proposition

A simple triangulation of bounded degree is $\chi-$complete.

Y. Chebbi
The discrete Laplacian of a 2-simplicial complex, accepted and reviewed in Potential Analysis, 26 p, (2017)

This assumption of $\chi-$completeness is in relationship with resembling techniques in the following papers :

J. Masamune
A Liouville Property and its Application to the Laplacian of an Infinite Graph,
Contempory Mathematics **484** , 103–115. (2009)

X. Huang, M. Keller, J. Masamune, R.K. Wojciechowski,
A note on self-adjoint extensions of the Laplacian on weighted graphs,
J. Funct. Anal. **265** no. 8, 1556–1578 (2013)

The definition of $\chi-$completeness is used later in the recent paper :

Hatem Baloudi, Sylvain Golenia, Aref Jeribi
The adjacency matrix and the discrete Laplacian acting on forms, arXiv :1505.06109 (2015)

and it is also mentioned in this book :

A. Jeribi
Spectral Theory and Applications of Linear Operators and Block Operator Matrices, Springer, (2015).

Essential self-adjointness

Theorem

If the connected locally finite graph is $\chi-$complete, then the Gauß-Bonnet operator D is essentially self-adjoint.

Corolary

If the connected locally finite graph is $\chi-$complete then the Laplacian Δ_0 is essentially self-adjoint.

Theorem

If the triangulation is $\chi-$complete then the Gauss-Bonnet operator T is essentially self-adjoint on $C_c^0(K) \oplus C_c^1(K) \oplus C_c^2(K)$.

Corolary

If the triangulation is χ-complete then the Helmholtzian $\mathcal{L}$ is essentially self-adjoint on $C_c^0(K) \oplus C_c^1(K) \oplus C_c^2(K)$.

Plan

Spectra of Δ_1 and Δ_0

We denote the spectrum of Δ_i by $\sigma(\Delta_{iff})$

Theorem

$$\sigma(\Delta_1) \setminus \{0\} = \sigma(\Delta_0) \setminus \{0\}.$$

Theorem

Let the graph be a connected weighted locally finite infinite graph such that the edge weight c is bounded as $\frac{1}{\alpha} \leq c(x, y) \leq \alpha$, for some α, for all edge (x, y). Then

$$0 \in \sigma(\Delta_1) \text{ or } 0 \in \sigma(\Delta_0).$$

H. Ayadi
Spectra of Laplacians on forms on an infinite graph, Operator and Matrices, 11 :2 , 567–586 (2017)

Plan

Preliminaries

Metrics

Discrete functions and forms

Operators on weighted graphs

Essential self-adjointness

Spectra

Activities of GS-Team from 2013 - - -

GS-Team

Graph & Spectra Team at Tunisia

Oneday-Workshop "Graphs & Spectra (GS-2013)"
Binzart ; June 28th, 2013

Oneday-Workshop "Geometry & Analysis on Graphs (GS-2014)"
Bizerte ; Mars 18th, 2014

Oneday-Workshop "Graphs & Spectra (GS-2015)"
Defense of PhD Thesis of Hèla Ayadi
Bizerte ; November 20th, 2015

Research School CIMPA-2016
"Théorie Spectrale des Graphes et des Variétés"
Kairouan ; November 7th—19th, 2016

Defense of the PhD Thesis of Marwa Balti
Bizerte ; May 20th, 2017

Many Thanks for your attention