Simplicial Complexes

Sheet 5

(1) Show that the simplicial complexes A_n, $n \in \mathbb{N}$ are chamber complexes (see example in lecture after Theorem 1.9). Give an example for a coloring.

(2) Show that the simplicial complexes C_n, $n \in \mathbb{N}$ are chamber complexes (see example in lecture after Theorem 1.9). Give an example for a coloring.

(3) (a) Let Δ and Δ' be incidence complexes and $\alpha : X_\Delta \to X_{\Delta'}$ an incidence preserving map, i.e. $x \neq y$ in X_D with $x I_\Delta y$ implies $\alpha(x) \neq \alpha(y)$ with $\alpha(x) I_\Delta \alpha(y)$. Show that α can be extended to a morphism $\widetilde{\alpha} : \Delta \to \Delta'$. Is this morphism uniquely determined? (b) Is the statement still valid if Δ, Δ' are simplicial complexes that are not necessarily incidence complexes?

(4) Let X be the system of subspaces of $\mathbb{R}^n$, $n \geq 1$. Give an example of a chamber morphism $\Delta(X, \leq) \to \Delta(X, \leq)$ that is not special.