Simplicial Complexes

Summer semester 2016 **Prof. Dr. M. Keller**

Sheet 4

(1) Is the poset of cosets $\Delta(G,S)$ of a finitely generated group G with a minimal generating set S that is a simplicial complex an incidence complex?

(2) Let $\Delta(X_1,I_1)$ and $\Delta(X_2,I_2)$ be incidence complexes. Show that $\Delta(X_1,I_1)\times\Delta(X_2,I_2)$ is an incidence complex, i.e., show that there is an incidence relation I such that

$$\Delta(X_1\times X_2, I)=\Delta(X_1,I_1)\times\Delta(X_2,I_2).$$

(3) Let $(\Delta,\leq)$ be an ordered set. What is the relation between $\Delta(X,\leq)$ and $\Delta(X,\geq)$?

(4) The hyper cube in $\mathbb{R}^n$ is given by the set

$$C:=\{(x_1,\dots,x_n)\in\mathbb{R}^n \mid -1\leq x_i\leq 1, i=1,\dots,n\}.$$

Let the faces of the cube be denoted by

$$X=\{C\cap H_i^+, C\cap H_i^- \mid i=1,\dots,n\}$$

where H_i^+ and H_i^-, $n=1,\dots,n$, are the hyper surfaces be given by

$$H_i^{\pm}:=\{(x_1,\dots,x_n)\in\mathbb{R}^n \mid x_i=\pm 1\}.$$

Show that $\Delta(X,\leq)$ is isomorphic to $C_n=\Delta(Y_n,\subseteq)$ with

$$Y_n:=\{S\subseteq\{\pm 1,\dots,\pm n\} \mid S\neq\emptyset,\ \{i,-i\}\not\subseteq S,\ i=1,\dots,n\}.$$