Simplicial Complexes

Summer semester 2016 **Prof. Dr. M. Keller**

Sheet 2

1. Let $G = \langle a, b, c \mid a^2 = b^2 = c^2 \rangle$ be the free group with three generators with $S = \{a, b, c\}$. Show that S is minimal. Give an explicit description of the cosets gG_J for $\emptyset \subseteq J \subseteq S$, $g \in G$. Which of these cosets coincide?

2. Let G be a group with generating set S. Show that S is minimal if and only if for all $s \in S$ and all $s_1, \ldots, s_n \in S \cup S^{-1}$ we have $s \neq s_1 \ldots s_n$ (where $S^{-1} = \{r^{-1} \mid r \in S\}$).

3. Let $(\Delta, \leq)$, $(\Delta', \leq)$ be two simplicial complexes. Show that $\Delta \times \Delta'$ with the product order

$$(x, x') \leq (y, y') \quad \Leftrightarrow \quad x \leq y,\ x' \leq y'$$

   is a simplicial complex.

4. Show that for $G = \mathbb{Z}^d$ the set $S = \{e_1, \ldots, e_d\}$, with e_k being the vector with all zeros except for the k-th component which has a one, $k = 1, \ldots, d$, is a minimal generating set and

$$\bigcap_{s \in S \setminus J} G_{S \setminus \{s\}} = G_J \text{ for all } J \subseteq S.$$