Simplicial Complexes

Summer semester 2016 **Prof. Dr. M. Keller**

Sheet 1

(1) Let $v_0, \ldots, v_n$, $n \in \mathbb{N}$, be elements of a vector space V over $\mathbb{R}$. Show that the following statements are equivalent:

   (a) $v_0, \ldots, v_n$ are affinely independent.

   (b) For all $\lambda_0, \ldots, \lambda_n \in \mathbb{R}$ such that $\lambda_0 v_0 + \ldots + \lambda_n v_n = 0$ and $\lambda_0 + \ldots + \lambda_n = 0$ we have $\lambda_1 = \ldots = \lambda_n = 0$.

   (c) The set $\{v_0, \ldots, v_n\}$ is included in no $(n-1)$-dimensional hypersurface of V. Any k-dimensional hypersurface H of V is given by a vector $v \in V$ and a k-dimensional subspace W of V via

$$H = v + W := \{v + w \mid w \in W\}.$$

(2) Let $B = \{v_0, \ldots, v_n\}$ be a set of affinely independent vectors in a real vector space V, let

$$F = \{\mathrm{conv}(W) \mid W \subseteq B\},$$

and

$$j : P(\{0, \ldots, n\}) \to F, W \mapsto \mathrm{conv}(W).$$

Show that j is an order isomorphism.

(3) Let G be a group with generating set S. Let For $\emptyset \neq J \subseteq S$ let $G_J = \langle J \rangle$ and $G_\emptyset = \{e\}$ with the neutral element e. Let $g \in G$. Show that the following statements are equivalent:

   (i) $g \in G_J$

   (ii) $gG_J = G_J$

   (iii) gG_J is a subgroup of G

(4) Let $G = \mathbb{Z}^2$ be given with generating set $S = \{(1,0), (0,1), (1,1)\}$. Give an explicit description of the cosets gG_J for $\emptyset \subseteq J \subseteq S$, $g \in \mathbb{Z}^2$. Which of these cosets coincide?

Optional Problems

(OP1) Consider B And j as in (2).

   (a) Give an example where j fails to be an isomorphism when the vectors in B are not affinely independent.

   (b) Give an example where j is still an isomorphism although the vectors in B are not affinely independent.

   (c) Find the a characteristic property of the vectors in B such that j is an isomorphism if and only if this property is fulfilled.