C*-Algebras

Winter semester 2016/17 **Prof. Dr. M. Keller**

Sheet 3

(1) Show that if an element in an algebra has a left inverse and a right inverse then these two inverses coincide. In particular, the inverse element in an algebra is unique.

(2) Show that if a, b in A satisfy $ab, ba \in GL(A)$, then $a, b \in GL(A)$.

(3) Let e be the n-dimensional unit matrix and $a \in \mathbb{C}^{n\times n}$, for some $n \in \mathbb{N}$. Show the following equivalence:

(i) a is invertible.

(ii) There is b such that $ba = e$ (or equivalently a is surjective).

(iii) There is b such that $ab = e$ (or equivalently a is injective).

(iv) 0 is not an eigenvalue of a.

(4) Show that the equivalence of (3) is general wrong for bounded linear operators on infinite dimensional spaces. (Hint: Show that there are $a, b \in \mathcal{L}(\ell^2(\mathbb{N}))$ such that $ab = e$ but a and b are not invertible.)