C*-Algebras

Winter semester 2016/17 **Prof. Dr. M. Keller**

Sheet 10

(1) Let $A = \ell^1(\mathbb{N}_0)$ be the Banach algebra with convolution as multiplication. Show that

$$\hat{A} = \mathbb{D} := \{z \in C \mid |z| \leq 1\}$$

and the Gelfand transform is given by

$$\Gamma : A \to C(\mathbb{D}), \quad \xi \mapsto \left(z \mapsto \sum_{n \geq 0} \xi_n z^n \right)$$

(2) Let A, B be C^*-algebras and $\phi : A \to B$ a homorphism of involutive algebras. Show that ϕ is continuous with norm $\|\phi\| \leq 1$. (Hint you can use that we have shown the corresponding statement for unital C^*-algebras in the lecture.)

(3) Let A, B be C^*-algebras and $\phi : A \to B$ an injective homorphism of involutive algebras. Show that ϕ is isometric. (Hint you can use that we have shown the corresponding statement for unital C^*-algebras in the lecture.)

(4) Let X be a locally compact Hausdorff space and μ a probability measure on X. Furthermore, for $f \in C_0(X)$, let M_f be the multiplication operator

$$M_f : L^2(X, \mu) \to L^2(X, \mu), \quad \xi \mapsto f\xi.$$

Show that M_f is bounded and

$$\sigma(M_f) = f(\text{supp}) \cup \{0\}.$$